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Endogenous fertility in a model with non-dynastic parental altruism

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Abstract. A model of fertility choice is studied in which the utility of parents depends on how much they consume, on how many children they have and on the consumption of their children. Hence, parents are altruistic towards their children, but in a more limited sense than in the much discussed dynastic fertility model presented by Becker and Barro (1988). The concept of a (subgame perfect) bequest equilibrium is used to solve the non-dynastic model considered here. The steady state birth rate is lower in the non-dynastic model than in the Becker-Barro model. However, the key qualitative predictions concerning the dynamic behavior of fertility are strikingly similar in both models.

JEL classification: J13, J11, D90.

Key words: Fertility, consumption, bequest equilibrium, altruism, non-dynastic preferences.

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1. Introduction

This paper studies an economic model of fertility choice in which the utility level of agents depends on their consumption, the number of children they

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have and on the consumption of their children. Hence, parents are altruistic towards their children, but in a more limited sense than in the much debated dynastic model of fertility choice recently presented by Becker and Barro (1988, henceforth BB) in which the utility of parents depends on the utility of their children and hence (indirectly) on the consumption and fertility decisions made by all subsequent generations.[1]

Non-dynastic preferences similar to those used here have widely been assumed in the growth literature, for example by Arrow (1973), Kohlberg (1976), Dasgupta (1974), Lane and Leininger (1984), Leininger (1986) and Bernheim and Ray (1987), but that research has abstracted from demographic issues (the number of children is not treated as a parental choice variable in that work). Apart from the non-dynastic utility function, the model here has the same structure as the BB model. The present paper focuses on how this change in preferences affects fertility behavior.

The dynastic (time-consistent) preferences assumed by BB allow to reduce their model to a single generation's optimization problem. This problem is solved by the head of a dynastic family who "acts as if he maximizes dynastic utility subject to a dynastic resource constraint" (BB, p. 23). By contrast, game theoretic equilibrium concepts are needed to solve the model with non-dynastic altruism, as in that model a conflict of interest exists between parents and children about how the latter should use the inheritance that they receive from their parents. The paper applies the notion of a (subgame perfect) bequest equilibrium due to Leininger (1986) and Bernheim and Ray (1987).[2]

Steady state fertility is lower when non-dynastic preferences are assumed. However, rather surprisingly, the non-dynastic model generates predictions concerning the dynamics of fertility (and of parental bequests to children) that are strikingly similar to those of the BB model. This seems noteworthy as BB stress that the dynastic preference specification is a central feature of their model (BB, p. 2).

Like the BB model, the non-dynastic model predicts that the steady state birth rate is an increasing function of the interest rate, and that it is negatively linked to the rate of technical progress. (BB argue that these predictions and the ones that follow are consistent with phenomena such as the low level of fertility observed in Western countries during the last decades and the baby boom after World War II.) In both models, a permanent increase in the cost of raising children (say, a permanent cut in a subsidy to child rearing) induces a transitory fall in the birth rate (however, permanent changes in the cost of child rearing have no long run effect on the birth rate); a temporary increase in the cost of raising children induces a temporary fall in the birth rate that is followed by a temporary increase.

Both models predict that a negative shock to the inherited wealth of a given generation (e.g., destruction of capital due to a war) induces that generation to have fewer children; however, the wealth shock has no effect on bequests per child made by that generation, and thus it does not affect the birth rates of subsequent generations.

Section 2 of the paper presents the non-dynastic model. Section 3 discusses model predictions.

2. The model

2.1 Preferences and budget constraints

The model is closely related to those of BB, Leininger (1986) and Bernheim and Ray (1987). An infinite sequence of generations i=0, 1, 2, ... is considered. All members of the same generation are identical. All agents live two periods; they consume and give birth to children during the second period of their life (no consumption takes place during the first period).

A member of generation i maximizes the following utility function:

$$u_i = c_i^\sigma + a(n_i)\, n_i\, c_{i+1}^\sigma, \tag{1}$$

where c_i and n_i are, respectively, the consumption of that agent and the number of children that the person has. The term $a(n_i)$ measures the degree of parental altruism towards each child. Following BB, $a(n_i) = a n_i^{-\varepsilon}$ is assumed. σ, ε and a are parameters that satisfy: $a>0$; $0<\sigma$, $\varepsilon<1$ and $\sigma+\varepsilon<1$. Thus, u_i is increasing and concave in c_i and in n_i; $\sigma+\varepsilon<1$ ensures that i's decision problem is well defined.

Generation i receives an inheritance from generation i–1; this inheritance can be used for consumption, in order to raise children and as a bequest. Parents cannot leave negative bequests. The budget constraint of a member of generation i is:

$$c_i + n_i(k_{i+1} + b_i) = k_i(1+r), \tag{2}$$

where k_{i+1} denotes the bequest per child made by that person. $b_i>0$ is the cost of raising one child. $r>0$ is the return on capital.[3]

Parents cannot dictate their children's consumption, but they can influence it through the bequest that they make. Generation i anticipates that c_{i+1} is a function of $k_{i+1}(1+r)$:

$$c_{i+1} = g_{i+1}(k_{i+1}(1+r)). \tag{3}$$

In contrast to (1), BB assume that the utility of generation i depends on the utility of each child and hence on the consumption and the birth rate of all subsequent generations:

$$u_i = c_i^\sigma + a n_i^{1-\varepsilon} u_{i+1} = c_i^\sigma + a n_i^{1-\varepsilon} c_{i+1}^\sigma + a^2 n_i^{1-\varepsilon} n_{i+1}^{1-\varepsilon} c_{i+2}^\sigma + \dots . \tag{4}$$

Note that the non-dynastic model considered in this paper only keeps the first two terms on the right-hand side of this utility function. The BB model can be solved by maximizing the dynastic utility function (4) of generation i=0 subject to the restriction that the budget constraint (2) holds for all $i \geq 0$. This yields the following first-order conditions: $1 = (1+r)\, a n_i^{-\varepsilon} (c_{i+1}/c_i)^{\sigma-1}$, $c_{i+1} = (\sigma/(1-\sigma-\varepsilon))\, b_i(1+r)$ for $i \geq 0$. Using these conditions, it is easy to verify the assertions about the BB model that are made at various points in the text.

2.2 Optimal behavior and equilibrium

Substitution of n_i from (2) and of c_{i+1} from (3) into (1) shows that generation i seeks to maximize the following objective function with respect to c_i and k_{i+1}:

$$u_i = c_i^{\sigma} + a\,(k_i\,(1+r) - c_i)^{1-\varepsilon}\,[g_{i+1}\,(k_{i+1}\,(1+r))]^{\sigma}/[k_{i+1} + b_i]^{1-\varepsilon}. \quad (5)$$

Clearly, the optimal value of k_{i+1} satisfies:

$$k^*_{i+1} \in \operatorname{Arg}\ \underset{k\geq 0}{\operatorname{Max}}\ [g_{i+1}\,(k\,(1+r))]^{\sigma}/[k+b_i]^{1-\varepsilon}.$$

Hence, it is seen immediately that the optimal per-child bequest made by generation i is unaffected by a change in its inherited wealth $(k_i\,(1+r))$.

Maximizing (5) with respect to c_i yields the following first-order condition:

$$c_i + c_i^{(1-\sigma)/\varepsilon}\,\Psi_i\,(g_{i+1}) = k_i\,(1+r),$$

with $\Psi_i\,(g_{i+1}) \equiv \{[a\,(1-\varepsilon)/\sigma]\,[g_{i+1}\,(k^*_{i+1}\,(1+r))]^{\sigma}/[k^*_{i+1} + b_i]^{1-\varepsilon}\}^{1/\varepsilon}$ (where k^*_{i+1} is defined above). Let

$$G_i\,(y|g_{i+1}) \equiv \{c|c + c^{(1-\sigma)/\varepsilon}\Psi_i\,(g_{i+1}) = y\}. \quad (6)$$

The function $c_i = G_i\,(y|g_{i+1})$ indicates how much generation i wishes to consume if its inheritance equals y, given that the consumption schedule of generation i+1 is g_{i+1}.

The concept of a (subgame perfect) bequest equilibrium proposed by Leininger (1986) and by Bernheim and Ray (1987) is applied. (As mentioned earlier, these authors too study dynamic models with non-dynastic parental altruism, but in their analysis the number of children is not treated as a parental choice variable.) Given an initial capital stock k_0 and a sequence of marginal child rearing costs $\{b_i\}_{i\geq 0}$, a bequest equilibrium is a sequence of consumption schedules, consumptions, bequests and birth rates $\{g^*_i, c^*_i, k^*_i, n^*_i\}_{i\geq 0}$ with the following properties:

(i) $g^*_i\,(y) = G_i\,(y|g^*_{i+1})$ for all $y \geq 0$ and for all $i \geq 0$.

(ii) $k^*_0 = k_0$ and $k^*_{i+1} \in \operatorname{Arg}\ \underset{k\geq 0}{\max}\ [g^*_{i+1}\,(k\,(1+r))]^{\sigma}/[k+b_i]^{1-\varepsilon}$ for all $i \geq 0$.

(iii) $c^*_i = g^*_i\,(k^*_i\,(1+r))$ and $n^*_i = [k^*_i\,(1+r) - c^*_i]/[k^*_{i+1} + b_i]$ for all $i \geq 0$.

Hence, generation i's equilibrium consumption schedule, g^*_i, reflects optimal consumption decisions by generation i, given that the consumption schedule of generation i+1 is g^*_{i+1} (point (i)). In equilibrium, generation i selects the per child bequest k^*_{i+1} that maximizes its own utility, given g^*_{i+1} (see (ii)). Generation i's wealth is $k^*_i\,(1+r)$, in equilibrium, and hence its consumption is $c_i^* = g_i^*(k_i^*(1+r))$; given c_i^*, k_i^* and k_{i+1}^*, i's equilibrium birth rate n_i^* is obtained from the budget constraint (2) (see (iii)).

Following Leininger (1986) and Bernheim and Ray (1987), the analysis here assumes that an agent's decisions depend only on actions taken by her parents, i.e. attention is restricted to Markov strategies. Economically, this assumption seems reasonable. Also, the equilibrium considered here would still be an equilibrium if agents were allowed to use more general strategies (cf. Leininger 1986; Fudenberg and Tirole 1991, Ch. 13).[4] The existence of an equilibrium is proved in the Appendix.

Let γ_i^* denote generation i's consumption to wealth ratio along the equilibrium path: $\gamma_i^* \equiv c_i^*/(k_i^*(1+r))$. A property of equilibrium that is important for the analysis below is that the sequence $\{\gamma_i^*\}_{i\geq 0}$ satisfies a first-order difference equation (see the Appendix for proofs of Eqs. (7)–(9)):

$$\gamma_i^* = f^i(\gamma_{i+1}^*) \text{ for } i \geq 0, \tag{7}$$

with

$$f^0(\gamma_1^*) \equiv h(\gamma_1^*, k_0, b_0) \text{ and } f^i(\gamma_{i+1}^*) \equiv s(\gamma_{i+1}^*, b_{i-1}/b_i) \text{ for } i \geq 1, \tag{8}$$

where h (.) and s (.) are differentiable functions. The equilibrium bequest k_{i+1}^* is linked to γ_{i+1}^*, as follows:

$$k_{i+1}^* = [\sigma\varepsilon/((1-\sigma-\varepsilon)(1-\varepsilon)]\, b_i/(1/(1-\varepsilon) - \gamma_{i+1}^*) \text{ for } i \geq 0. \tag{9}$$

3. Implications of the non-dynastic model

3.1 Steady states

When the marginal cost of raising children (b) grows at a constant rate, there exists a unique steady state. In steady state, the birth rate is constant, while consumption and bequests grow at the same rate as b. It can be verified that $n^* = [a(1+r)\varphi_b^{\sigma-1} - n^* a\varphi_b^{\sigma}(1-\varepsilon)(1-\sigma)/(\varepsilon\sigma)]^{1/\varepsilon}$ holds in the non-dynastic model, where n^* is the steady state birth rate, while $\varphi_b \equiv b_{i+1}/b_i$ is the growth factor of b. In the BB model, in contrast, $n^* = [a(1+r)\varphi_b^{\sigma-1}]^{1/\varepsilon}$ (see the summary of the BB model at the end of Sect. 2.1). The steady state birth rate is thus lower in the non-dynastic model. This is not surprising, as parents are less altruistic in the non-dynastic model than in the BB model.

Both models predict that the steady state birth rate is positively linked to the interest rate and to the altruism parameter a. Note that n^* depends on the growth rate of the marginal child rearing cost – but not on the level of that cost (b), per se; specifically, the steady state birth rate is a decreasing function of the growth rate of b. Steady state growth in the cost b can be due to technical progress that induces growth in the productivity of labor in the production of physical goods and, hence, in the opportunity cost to parents of raising children. Like the BB model, the model here predicts, thus, that fertility is negatively related to the rate of technical progress.

3.2 Dynamic effects of exogenous shocks

In what follows, the dynamic effects of a wealth change (represented by a shift in initial assets k_0) and of permanent and temporary changes in the marginal cost of raising children are studied. For these exogenous changes, the predicted response of fertility is qualitatively the same in the non-dynastic model and in the BB model. In particular, the two models predict that a fall in k_0 and a permanent increase in child rearing costs (a rise of b_i by the same proportion for all $i \geq 0$) both induce a fall in the birth rate of the initial generation, but that these two types of shocks have no effect on birth rates in subsequent generations. In both models, an increase in the child rearing cost of the initial generation b_0 (while b_i stays constant for $i \geq 1$) reduces n_0 and it raises n_1 (the birth rates of generations $i \geq 2$ are unaffected).

The following analysis assumes that equilibrium in the non-dynastic model is unique. (A sufficient condition that ensures uniqueness of the equilibrium is provided in the Appendix.) This assumption ensures that there exists a unique sequence $\{\gamma_i^*\}_{i \geq 0}$ that satisfies (7). The exogenous shocks discussed here have in common that they do not alter the function f^i for generations $i \geq q$, where $q = 1$ or $q = 2$ (see (7)). Uniqueness of equilibrium implies thus that the equilibrium consumption to wealth ratios (γ_i^*) of generations $i \geq q$ are unaffected by the exogenous changes discussed here[5].

3.2.1 A wealth change. Note from (8) that a change in k_0 does not alter the function f^i for $i \geq 1$. The consumption to wealth ratios γ_i^* of generations $i \geq 1$ are thus unaffected by a change in k_0. This implies (see (9)) that neither generation $i = 0$, nor subsequent generations modify the bequests that they give to each child (this is consistent with the finding in Sect. 2.2 that a change in the wealth of a given generation has no effect on the per child bequest made by that generation). A reduction (say) in k_0 merely induces generation $i = 0$ to consume less and to have fewer children. As the per capita wealth of generations $i \geq 1$ does not change when k_0 varies, the birth rates (and other decision variables) of generations $i \geq 1$ are unaffected by a change in k_0.

3.2.2 A permanent increase in the marginal cost of raising children. Assume that for all generations $i \geq 0$ the marginal cost of child rearing increases by the same proportion. From (8) it can be seen that such a change does not affect the function f^i for generations $i \geq 1$. Thus, the consumption to wealth ratios of generations $i \geq 1$ do not change; hence the per capita wealth of generations $i \geq 1$ and the per capita consumption of these generations rise by the same percentage as the increase in the child rearing cost (see (9)). The budget constraints of generations $i \geq 1$ imply thus that the birth rates of these generations are unaffected by a permanent equiproportional rise in b. A permanent increase in b does, however, lower the birth rate of the initial generation (i=0).

To see why n_0^* falls, note that the function $\gamma_0^* = f^0(\gamma_1^*) = h(\gamma_1^*, k_0, b_0)$ is increasing in b_0 (see (12) in Appendix; note there that $\sigma + \varepsilon < 1$, by assumption). Thus, a permanent increase in b raises γ_0^* and, hence, it increases the consumption of the initial generation. Also, (9) implies that generation i=0 increases the bequest that it gives to each child. From the budget constraint of generation $i=0$, it follows thus that n_0^* has to fall.

3.2.3 A temporary increase in the marginal cost of raising children. Assume that the child rearing cost in the initial generation (b_0) rises (while b_i stays constant for $i \geq 1$). This induces a fall in the birth rate of generation $i=0$ and a rise in the birth rate of generation $i=1$. n_1^* increases because the increase in b_0 raises the bequest that generation $i=0$ gives to each of its children. Bequests per child made by generation $i=1$ however do not change and hence the per capita wealth and the birth rates of generations $i \geq 2$ are unaffected.[6]

It can also be shown that an anticipated rise in b_j for $j>0$ lowers n_j and increases n_{j+1}, under the assumption that $\gamma_j^* = \gamma_{j+1}^*$ would hold in the absence of the change in b_j. However, such an assumption is not needed to get the response to a change in b_0 described above. A detailed discussion of the effects of a change in b is provided in a technical supplement to the paper that is available from the author.

Appendix

- *Derivation of Eq. (7)*

From the definition of equilibrium:

$$g_{i+1}^*(y) = \{c | c + c^{(1-\sigma)/\varepsilon}\, \Psi(g_{i+2}^*) = y\}, \tag{10}$$

where $\Psi(g_{i+2}^*) = \{(a(1-\varepsilon)/\sigma)[c_{i+2}^*]^\sigma/[k_{i+2}^* + b_{i+i}]^{1-\varepsilon}\}^{1/\varepsilon}$. Maximization of $[g_{i+1}^*(k(1+r))]^\sigma/[k+b_i]^{1-\varepsilon}$ with respect to k (see point (ii) in definition of equilibrium) shows that k_{i+1}^* satisfies the following first-order condition:

$$\begin{aligned}(\sigma/(1-\varepsilon))(k_{i+1}^* + b_i)(1+r) =& c_{i+1}^* + ((1-\sigma)/\varepsilon)(c_{i+1}^*)^{(1-\sigma)/\varepsilon} \\ & \cdot \Psi_i(g_{i+2}^*) \text{ for } i \geq 0.\end{aligned} \tag{11}$$

(11) and the equilibrium condition $c_i^* = g_i^*(k_i^*(1+r))$ yield (9) in the text. (9) and the condition $c_i^* + \Psi_i(g_{i+1}^*)(c_i^*)^{(1-\sigma)/\varepsilon} = k_i^*(1+r)$ allow to get (after simple but tedious steps):

$$1 = \gamma_0^* + (\gamma_0^*)^{(1-\sigma)/\varepsilon}(k_0)^{(1-\sigma-\varepsilon)/\varepsilon} A(\gamma_1^*)\, B(\gamma_1^*)\, b_0^{(\sigma+\varepsilon-1)/\varepsilon} J_0 \tag{12}$$

and

$$1 = \gamma_i^* + (\gamma_i^*)^{(1-\sigma)/\varepsilon} A(\gamma_i^*)^{-1} A(\gamma_{i+1}^*)\, B(\gamma_{i+1}^*)\, (b_{i-1}/b_i)^{(1-\sigma-\varepsilon)/\varepsilon} J \text{ for i} \geq 1. \tag{13}$$

Here, $A(\gamma) \equiv (1/(1-\varepsilon) - \gamma)^{(1-\sigma-\varepsilon)\varepsilon}$ and $B(\gamma) \equiv \gamma^{\sigma/\varepsilon}((1-\sigma)/(1-\sigma-\varepsilon) - \gamma)^{1-1/\varepsilon}$ are functions of γ, while J_0, $J>0$ are constants.

(12) and (13) implicitly define the functions $\gamma_0^* = f^0(\gamma_1^*) \equiv h(\gamma_1^*, k_0, b_0)$ and $\gamma_i^* = f^i(\gamma_{i+1}^*) \equiv s(\gamma_{i+1}^*, b_{i-1}/b_i)$ for $i \geq 1$ (see (7), (8)).

- *Existence and uniqueness of equilibrium*

To solve for an equilibrium, a sequence $\{\gamma_i^*\}_{i\geq 0}$ has to be found that satisfies (7). Once such a sequence has been found, $\{g_i^*, c_i^*, k_i^*, n_i^*\}_{i\geq 0}$ can easily be determined.

For $j \geq 1$, define the function $F^j(\gamma) \equiv f^0(f^1(...(f^{j-1}(\gamma))...)$ and the set $S^j \equiv \{F^j(\gamma) | 0 \leq \gamma \leq 1\}$ for $j \geq 1$. S^j is a closed interval. Note that $S^{j+1} = \{F^j(f^j(\gamma)) | 0 \leq \gamma \leq 1\} \subseteq S^j$ (N.B. $0 \leq f^j \leq 1$, as $0 \leq c_j^* \leq k_j^*(1+r)$); thus $S \equiv \bigcap_{j\geq 0} S^j$ is non-empty. A sequence $\{\gamma_i^*\}_{i\geq 0}$ that satisfies (7) can be constructed by choosing $\gamma_0^* \in S$ and by selecting γ_i^* for $i \geq 1$ ... using $\gamma_0^* = f^0(\gamma_1^*)$, $\gamma_1^* = f^1(\gamma_2^*)$, etc. The fact that the set S is non-empty therefore guarantees the existence of an equilibrium.

Equilibrium is unique if and only if the set S is a singleton. It can be verified that a sufficient condition for uniqueness is that $\sigma \leq 0.5$ and that there exists a number q such that b_i/b_{i-1} is constant for all $i \geq q$ (the proof of this statement is presented in a technical supplement available from the author).

Endnotes

1 Analyses of fertility choice in a dynastic framework can also be found in papers by, among others, Razin and Ben-Zion (1974), Barro and Becker (1989), Pestieau (1989), Becker et al. (1990), Benhabib and Nishimura (1989), Alvarez (1994) and Cigno and Rosati (1996).

2 Raut (1992) also studies subgame perfect equilibria in a non-dynastic fertility model. The use of a model close to BB's setup allows the present paper to fully characterize the dynamics of fertility; in contrast, Raut uses a more general framework and is merely able to analyze steady states. I learnt about Raut (1992) after the present research was completed. The paper here is thus a complementary and independent analysis.

3 The budget constraint (2) follows the one used by BB, except that they assume $c_i + n_i(k_{i+1} + b_i) = w_i + (1+r)k_i$, where w_i is i's wage income (BB also allow for a variable interest rate). But note that this can be expressed as: $c_i + n_i(k'_{i+1} + b'_i) = (1+r)k'_i$, where $k'_i \equiv w_i/(1+r) + k_i$, $b'_i \equiv b_i - w_{i+1}/(1+r)$. Human capital is thus subsumed in the variable k_i in (2).

4 Some readers of the paper have suggested to consider strategies of the form $c_{i-1} = g_{i+1}(k_{i+1}(1+r), n_i)$, rather than (3). When g_{i+1} is of that form, then n_{i-1} does not affect the decision problem faced by generation i; thus, the equilibrium schedule g_i^* would effectively only have $k_i^*(1+r)$ as an argument.

5 Even if the equilibrium is not unique, there exists an equilibrium after the change in the exogenous variables in which the consumption to wealth ratios for generations $i \geq q$ are the same as before the change (and hence the analysis presented below remains valid). To see this, note that the set of sequences of equilibrium consumption ratios for generations $i \geq q$ is given by $\{\{\gamma_i^*\}_{i\geq q} | \gamma_i^* = f^i(\gamma_{i+1}^*)$ for all $i \geq q\}$. This set does not change when f_i for $i \geq q$ does not change.

6 Note that the function f^i for $i \geq 2$ is unaffected by the change in b_0, which implies that γ_i^* for $i \geq 2$ does not change. Hence, k_i^* and c_i^* for $i \geq 2$ are unaffected, which explains why n_i^* for $i \geq 2$ does not change.

References

Alvarez F (1994) Two Essays in the Economic Theory of Fertility. Ph.D. Dissertation, Economics Department, University of Minnesota

Arrow K (1973) Rawls's Principle of Just Saving. *Swedish Journal of Economics* 75:323–335

Barro R, Becker G (1989) Fertility Choice in a Model of Economic Growth. *Econometrica* 57:481–501

Becker G, Barro R (1988) A Reformulation of the Economic Theory of Fertility. *Quarterly Journal of Economics* 103:1–25

Becker G, Murphy K, Tamura R (1990) Human Capital, Fertility and Economic Growth. *Journal of Political Economy* 98:S 12–S 37

Benhabib J, Nishimura K (1989) Endogenous Fluctuations in the Barro-Becker Theory of Fertility. In: Wenig A, Zimmermann KF (eds) *Demographic Change and Economic Development*. Springer, Berlin Heidelberg New York, 29–41

Bernheim D, Ray D (1987) Economic Growth with Intergenerational Altruism. *Review of Economic Studies* 54:227–243

Cigno A, Rosati F (1996) Jointly Determined Saving and Fertility Behaviour: Theory, and Estimates for Germany, Italy, UK and USA. *European Economic Review* (forthcoming)

Dasgupta P (1974) On Some Alternative Criteria for Justice Between Generations. *Journal of Public Economics* 3:405–423

Fudenberg D, Tirole J (1991) *Game Theory*. MIT Press, Cambridge, MA

Kohlberg E (1976) A Model of Economic Growth with Altruism Between Generations. *Journal of Economic Theory* 13:1–13

Lane J, Leininger W (1984) Differentiable Nash Equilibria in Altruistic Economies. *Zeitschrift für Nationalökonomie* (Journal of Economics) 44:329–347

Leininger W (1986) The Existence of Perfect Equilibria in a Model of Growth with Altruism Between Generations. *Review of Economic Studies* 53:349–367

Pestieau P (1989) Endogenous Fertility and Fixed Input in a Growth Model With Altruism. In: Wenig A, Zimmermann KF (eds) *Demographic Change and Economic Development*. Springer, Berlin Heidelberg New York, 29–41

Raut L (1992) Subgame Perfect Manipulation of Children by Overlapping Generations of Agents With Two-Sided Altruism and Endogenous Fertility. Working Paper 92-54, Economics Department, University of California, San Diego

Razin A, Ben-Zion U (1974) An Intergenerational Model of Population Growth. *American Economic Review* 65:923–933