Online Appendix: *Persistent Global Growth Differences and Euro Area Adjustment: Real Activity, Trade and the Real Exchange Rate*

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Abstract

This Online Appendix contains supplemental material for *Persistent Global Growth Differences and Euro Area Adjustment: Real Activity, Trade and the Real Exchange Rate.*

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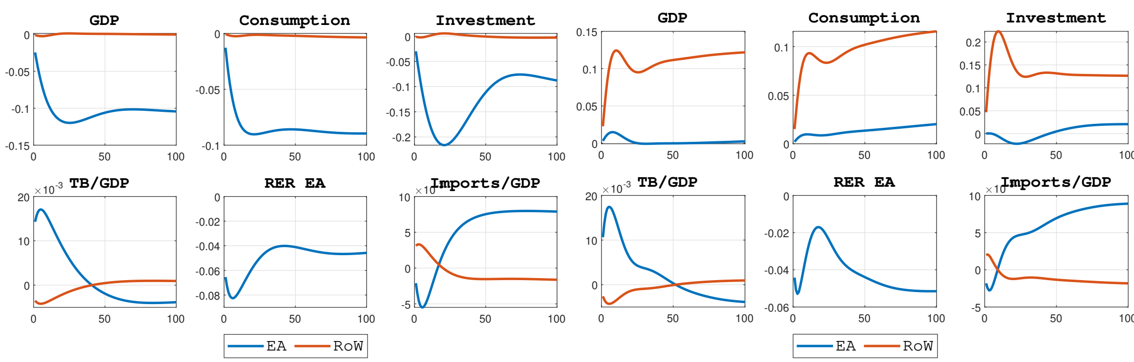
Contents

A Additional Results

This appendix presents additional historical shock decompositions for key EA and RoW macroeconomic variables. Specifically, Section A.1 reports the transmission mechanism of transitory productivity shocks and decompositions for EA consumption, EA investment, RoW GDP, and EA GDP growth (at annual frequency). Section A.2 provides decompositions of the EA trade-balance-to-GDP ratio and the EA real exchange rate, focusing on the contribution of demand shocks.

A.1 Additional impulse response functions



(a) IRF to transitory productivity shock EA (negative)

(b) IRF to transitory productivity shock RoW

Figure A.1: Impulse-response functions to temporary productivity shocks

A.2 Additional shock decompositions

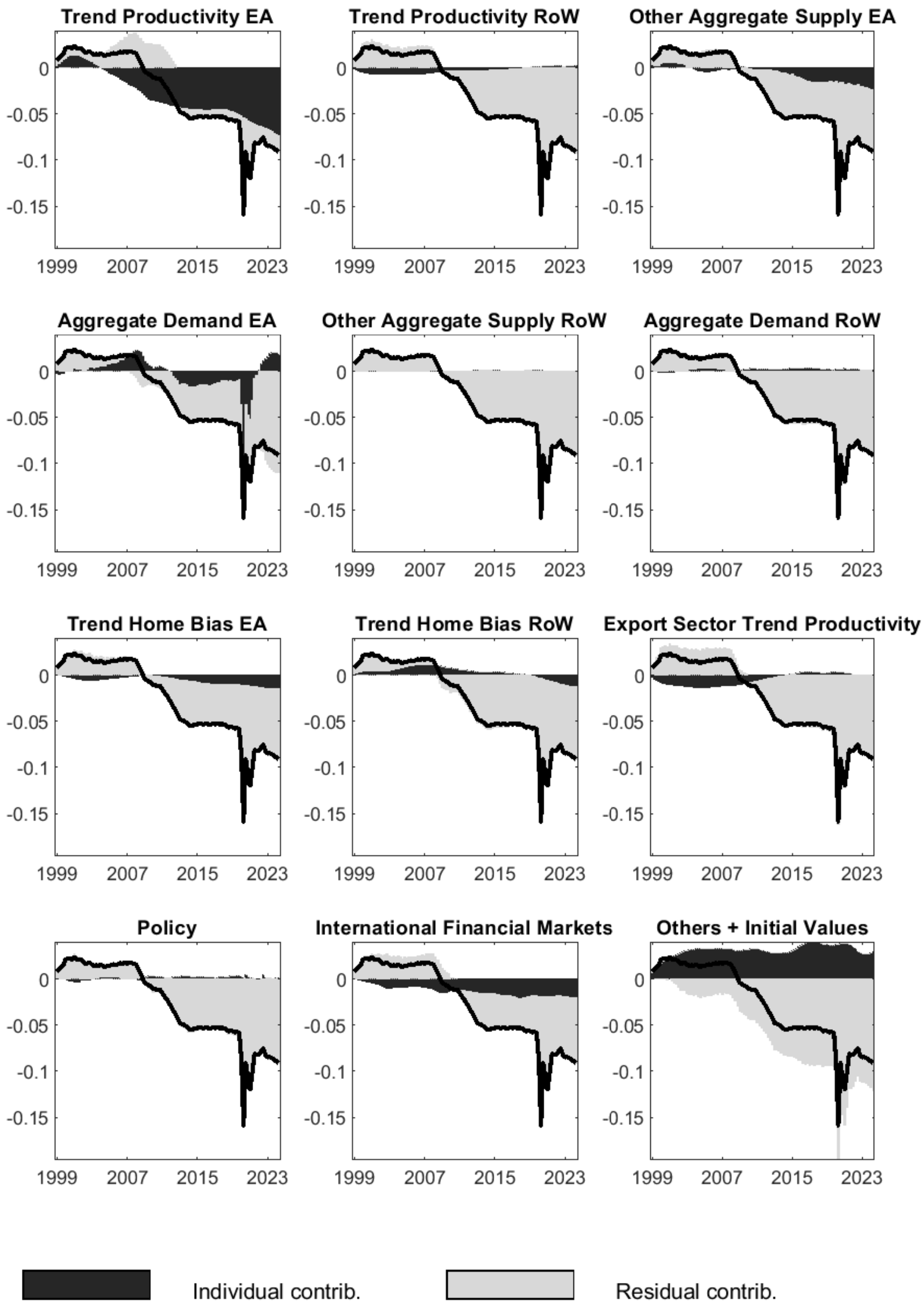


Figure A.2: Historical shock decomposition of EA real consumption (quarter-on-quarter)

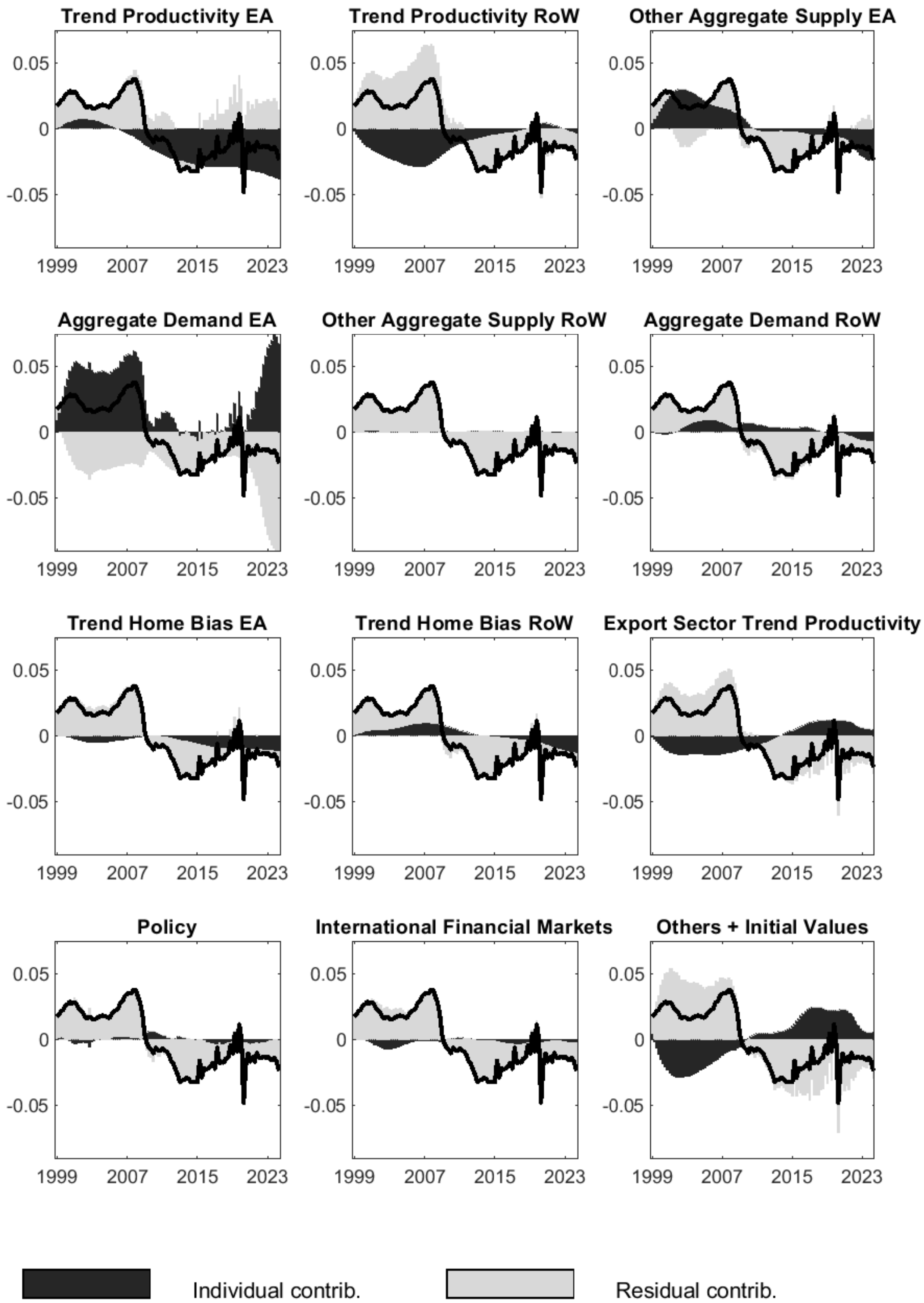


Figure A.3: Historical shock decomposition of EA real investment (quarter-on-quarter)

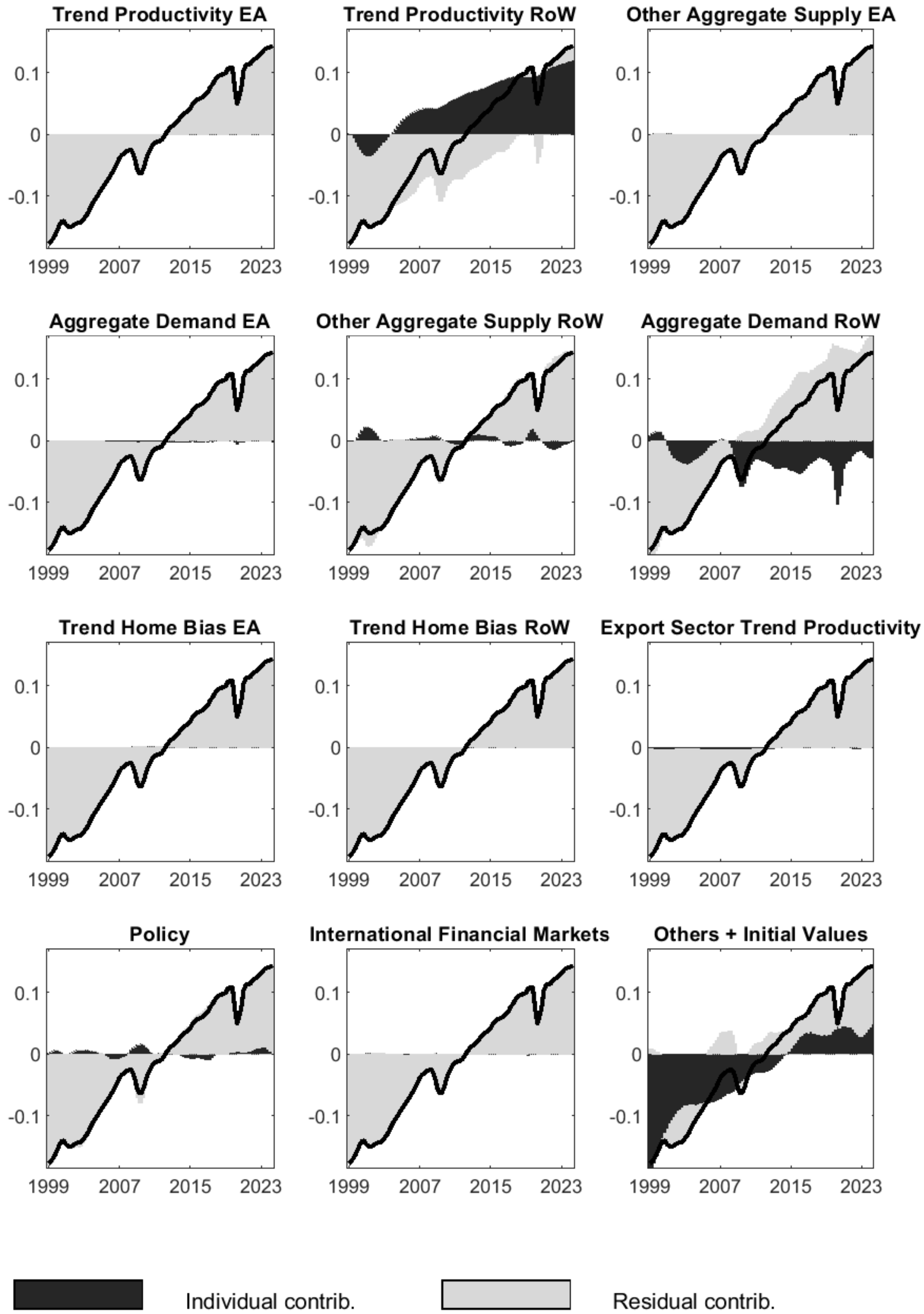


Figure A.4: Historical shock decomposition of RoW real GDP (quarter-on-quarter)

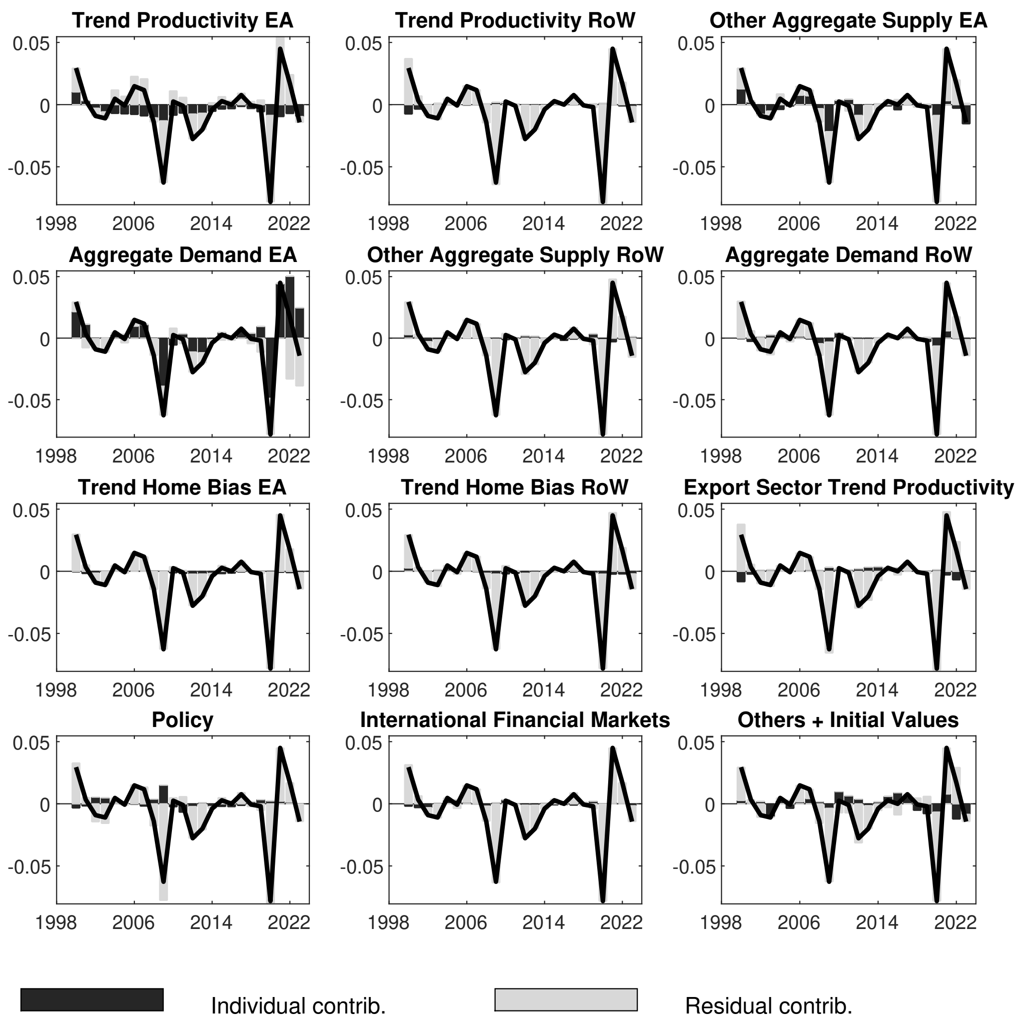
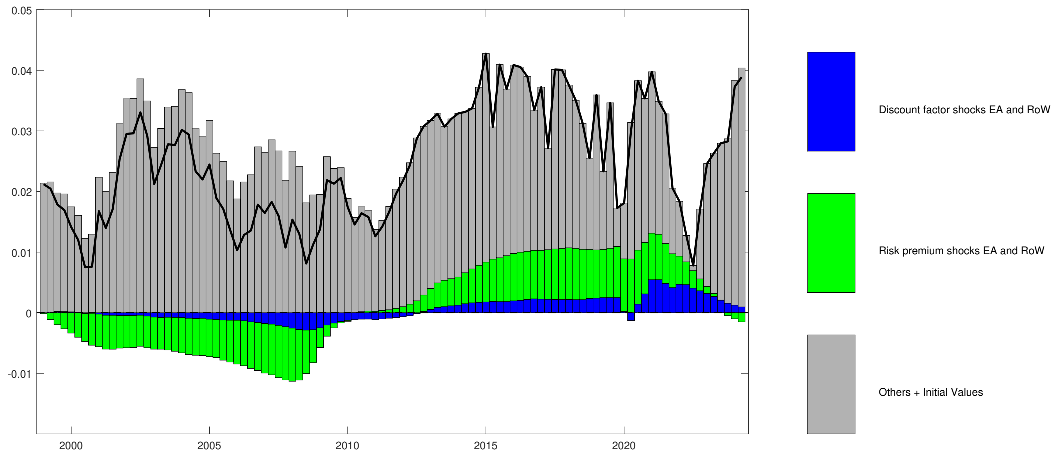
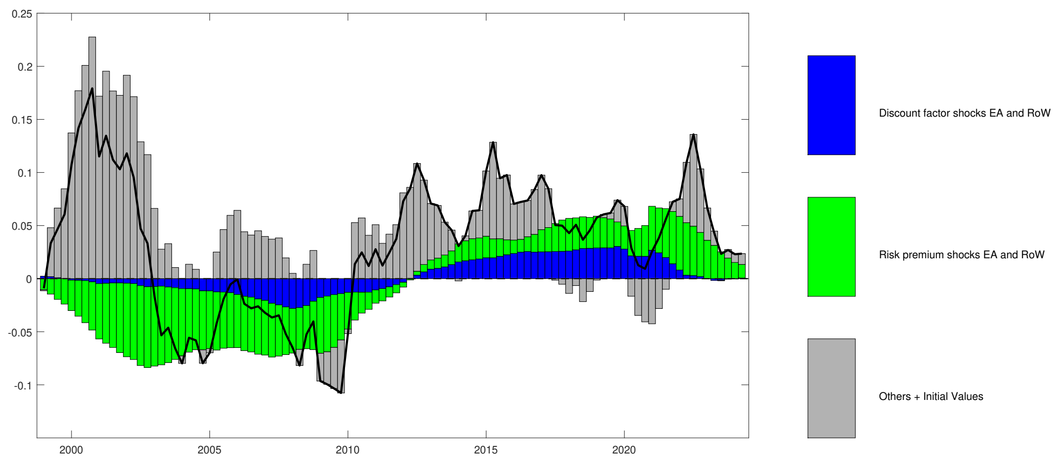


Figure A.5: Historical shock decomposition of EA real GDP growth (annual rate)



(a) EA trade-balance-to-GDP ratio (quarter-on-quarter)



(b) EA real exchange rate (quarter-on-quarter)

Figure A.6: Historical shock decompositions: contribution of demand shocks

B The core idea in the canonical IRBC model

This appendix presents a stripped-down two-country real business cycle model to illustrate the core mechanism linking trend growth rate shocks to trade balance and RER dynamics.

B.1 Model setup

We report impulse responses to a negative EA trend growth rate shock and a positive EA foreign-bias shock (i.e. negative home-bias shock) in the canonical two-country real business cycle (RBC) model of Backus et al. (1994) [BKK] (see Section 3.1 of the main text). The specification and the technology and preference parameters are identical to those in the benchmark model variant of BKK (subjective discount factor 0.99, risk aversion 2, labor supply elasticity 2.25, trade elasticity 1.5, and capital depreciation rate 0.025). The results are robust to parameter changes around these baseline values. The simulations reported below set the steady-state trend growth rate to zero, following BKK. Assuming a positive steady-state trend growth rate (as in our estimated model) does not materially affect the results.

Our "simple" international RBC model differs from the BKK model in only three dimensions:

(i) We allow for countries of different size, to match the EA-versus-RoW calibration in our estimated model. The smaller country, corresponding to the EA, is assumed to account for 18% of world output in steady state.

(ii) The only internationally traded asset is a real, non-state-contingent bond.

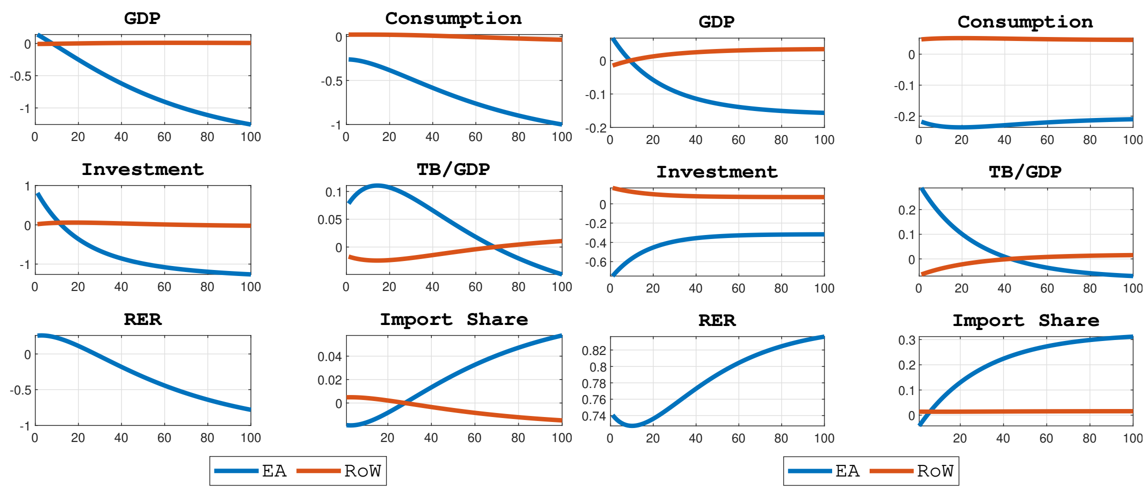
(iii) The TFP shock process is identical to that in our estimated baseline model, i.e. it includes productivity trend growth rate shocks. We also allow for shocks to the foreign-bias term in the final-good aggregator, using the estimated process.

By contrast, BKK assume two equal-sized countries, complete international financial markets, and they postulate a stationary productivity process (while abstracting from foreign bias shocks).

The complete-markets model has counterfactual implications (e.g. it predicts a counterfactual link between the real exchange rate and relative domestic/foreign consumption; Kollmann (1991, 1995)). Estimates of versions of the larger model with complete and incomplete markets, respectively, clearly favor the bonds-only variant over the complete-markets variant, in a statistical sense. The complete markets BKK model predicts that a negative EA trend growth shock generates a persistent deterioration of the EA trade balance.

B.2 Impulse responses

The panels in Figure B.1 show dynamic responses to negative one-standard-deviation innovation to the EA trend TFP growth rate and to EA trend foreign bias growth trend, respectively, for our variant of the BKK model.



(a) IRF to trend growth rate shock EA (negative)

(b) IRF to trend home bias EA

Figure B.1: Impulse-response functions in a textbook IRBC model

The BKK-style model predicts much more pronounced impact responses of consumption and investment to productivity trend growth rate shocks than the estimated model. A negative EA trend growth rate shock triggers a sharp immediate fall in consumption, which raises EA labor supply and thereby induces an increase in EA GDP and investment on impact. In the BKK-style economy, the EA trade balance improves on impact and remains above its unshocked value for nearly 70 quarters. The resulting increase in the trade balance is therefore long-lasting, though slightly less persistent than in our estimated model. The EA RER depreciates on impact, owing to the strong contraction in EA consumption demand. In subsequent periods, the EA RER follows a steady appreciation trend.

A negative EA home-bias shock likewise triggers a strong immediate contraction in EA consumption and investment. The EA trade balance improves sharply on impact, and then returns quickly to its unshocked value. By contrast, the EA RER exhibits a highly persistent depreciation, as in the estimated model.

In sum, the simple model predicts that a negative productivity trend growth rate shock has a long-lasting positive effect on the EA trade balance and induces a trend appreciation of the EA RER. An EA foreign bias shock, by contrast, generates a persistent depreciation of the EA RER. EA foreign bias shocks therefore have the potential to offset the appreciation trend induced by negative EA trend growth shocks.

The richer estimated model incorporates adjustment frictions in consumption, investment, and trade flows, and the estimates indicate that these frictions are quantitatively important. As a result, the estimated model implies that trend growth shocks generate a more gradual, and hence more persistent, adjustment of the trade balance. Moreover, because the estimated model allows for a broad set of alternative disturbances, it can identify more clearly the distinct role of trend growth shocks in driving the trade balance and other macroeconomic variables.

C Robustness

This appendix assesses the robustness of our main results to alternative assumptions and model variants: (i) endogenous euro-area productivity trend growth, modeled through a reduced-form link to risk premia shocks and the fiscal stance (Section C.1); (ii) a near-random-walk specification for labor productivity growth (Section C.2); (iii) nominal rigidities and household heterogeneity (Section C.3); (iv) export price setting, pass-through, and the terms of trade (Section C.4); and (v) imperfect substitutability between domestic and foreign goods (Section C.5).

C.1 Endogenous euro-area trend productivity growth

The baseline model treats productivity trend growth as exogenous. However, recent work (e.g., Benigno et al. (2025)) emphasizes that productivity growth may itself respond to macroeconomic conditions, for instance through R&D investment, sectoral reallocation, or demand-driven innovation. In particular, weak demand, fiscal austerity, or adverse financial conditions may depress investment and thereby slow productivity growth over time.

To explore this possibility within our framework, we introduce a reduced-form feedback from macroeconomic conditions to EA trend productivity growth. Specifically, we modify the EA productivity growth process as:

$$g_t^{EA} = \rho^{EA} g_{t-1}^{EA} + (1-\rho^{EA})\bar{g}^{EA} + \varepsilon_t^{EA} + \alpha_X^{EA}\big(X_t^{EA} - \rho_X X_{t-1}^{EA}\big), \tag{C.1}$$

where X_t^{EA} captures macroeconomic conditions that may influence innovation. The feedback operates through the moving average term $X_t^{EA} - \rho_X X_{t-1}^{EA}$. The exercise should therefore be interpreted as a reduced-form test of whether unexpected deterioration in financial or fiscal conditions help explain the observed productivity slowdown, not as a comprehensive model of endogenous innovation.

We consider two alternatives for X_t^{EA}: (i) the investment risk premium, proxying financial conditions and investment wedges; and (ii) a measure of the fiscal stance, capturing demand-driven effects.

In the first case, X_t^{EA} corresponds to the investment risk premium, which follows an autoregressive process. The specification in equation (C.1) therefore implies that trend productivity growth responds to the (filtered) risk-premium process (captured by the term $X_t^{EA} - \rho_X X_{t-1}^{EA}$). Economically, this captures the idea that unexpected changes in financial conditions, such as shifts in financing costs or risk perceptions, affect investment and innovation decisions.

In the second case, we proxy the fiscal stance using discretionary fiscal effort (DFE) as defined by the European Commission (2013):

$$\mathrm{DFE}_t = \frac{R_t^G}{Y_t} - \frac{\Delta E_t^G - (\Delta Y_t^{pot} - 1)E_{t-1}^G}{Y_t}, \tag{C.2}$$

where E_t^G denotes adjusted nominal government expenditure, R_t^G denotes nominal revenues, and Y_t^{pot} is medium-term nominal potential output. The total nominal government expenditure is defined as

$$E_t^G = P_t^G G_t + P_t^{IG} I_t^G + P_t^Y T_t. \tag{C.3}$$

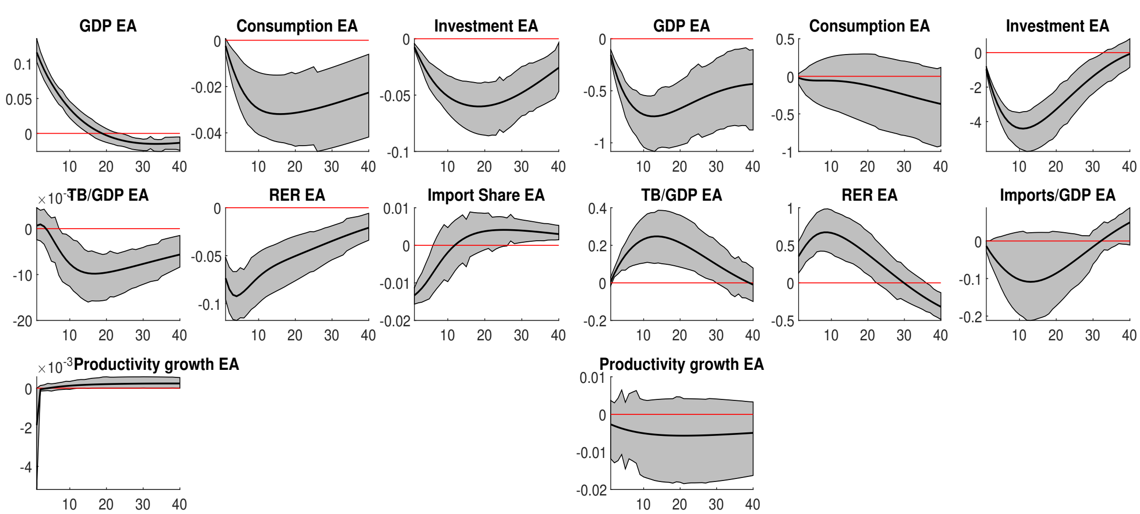
The specification in (C.1) does not introduce a microfounded innovation sector. Rather, it allows productivity growth to respond in reduced form to shocks already present in the model. The exercise can therefore be interpreted as testing whether part of the observed EA productivity slowdown is endogenously induced by demand or financial disturbances.

An important identification margin is the pre-crisis boom. If looser financial conditions stimulated innovation, the period of low risk premia and strong investment before the global financial crisis should have been associated with higher trend productivity growth. In the data, however, EA trend productivity growth was already weak during that period. Consistent with this observation, the estimated feedback coefficients are small and statistically insignificant, and the main trade-balance and exchange-rate decompositions remain qualitatively unchanged.

Figures C.3-C.6 report the results from re-estimating the model under these two alternatives. The estimated feedback coefficients α_X^{EA} are quantitatively small, and zero lies well within their highest posterior density intervals (see Figure C.2).

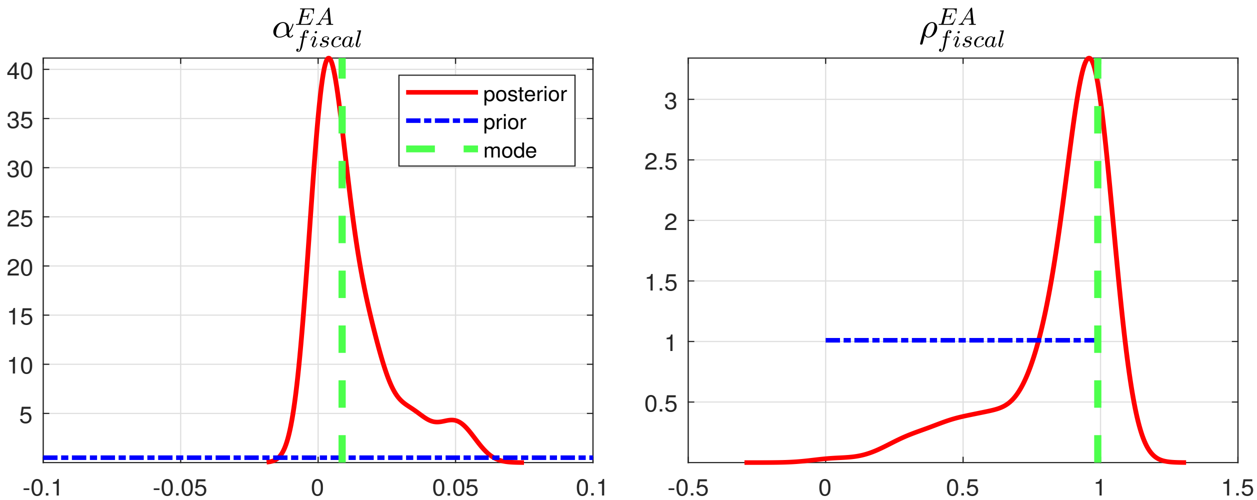
Moreover, the historical decompositions of the trade balance and the real exchange rate remain qualitatively unchanged. Figures C.3–C.6 report the corresponding historical decompositions with posterior uncertainty bands. While the point estimates suggest only a limited role for endogenous productivity growth, the uncertainty surrounding the contribution of TFP shocks is sizable. In particular, the wide bands indicate that the data are only weakly informative about the magnitude of the feedback from macroeconomic conditions to trend productivity growth.

These results suggest that, within our sample and model structure, the EA productivity slowdown is not primarily driven by endogenous responses to demand or financial shocks captured by the model. Allowing productivity growth to respond to these macroeconomic conditions does not overturn the interpretation that the EA productivity slowdown reflects persistent structural forces rather than endogenous demand-driven innovation effects.

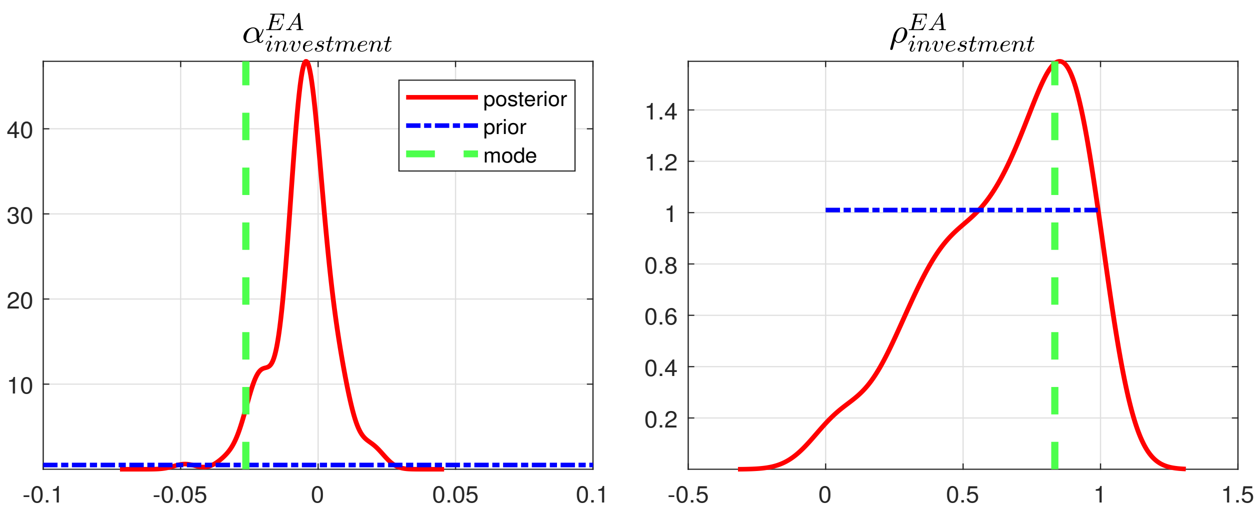


(a) IRF to gov. spending shock (b) IRF to risk premium shock

Figure C.1: Endogenous trend growth impulse response functions: fiscal stance (panel a) and risk premium (panel b)



(a) Fiscal stance



(b) Risk premium

Figure C.2: Prior/posterior distributions of the elasticity α^{EA}_X and persistence ρ^{EA}_X parameters in equation C.1: fiscal stance (panel a) and risk premium (panel b). The estimation is performed using four MCMC chains; the posterior is computed across all chains while the point estimate corresponds to the posterior mode.

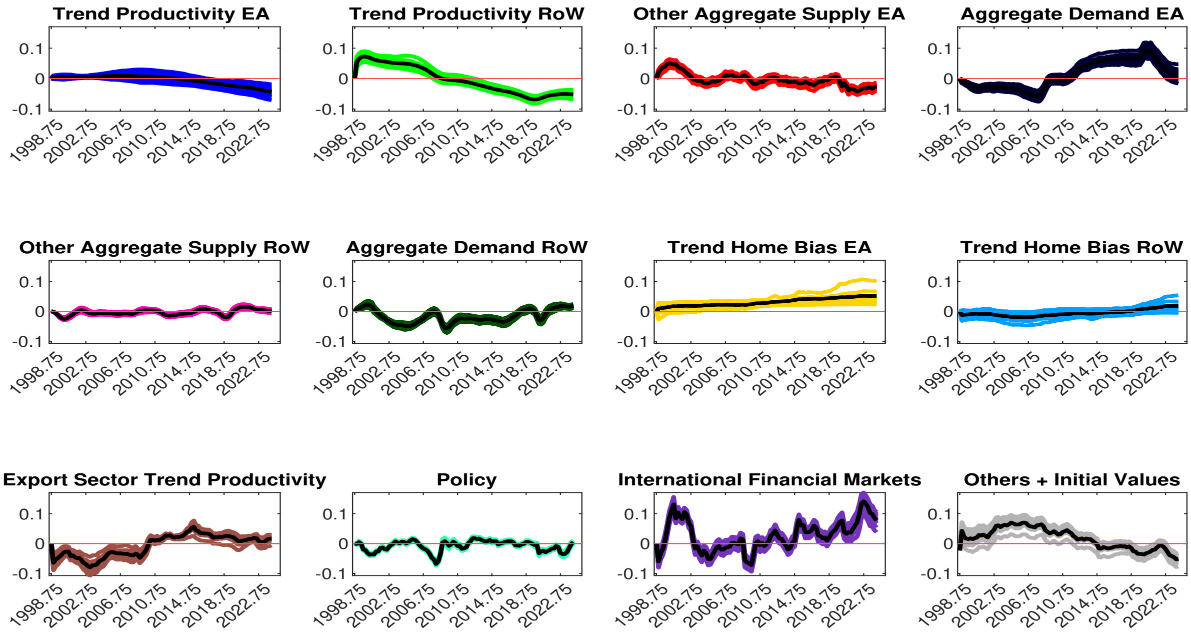


Figure C.3: Endogenous trend growth (risk premium): historical decomposition of EA Real Exchange Rate with parameter uncertainty bands

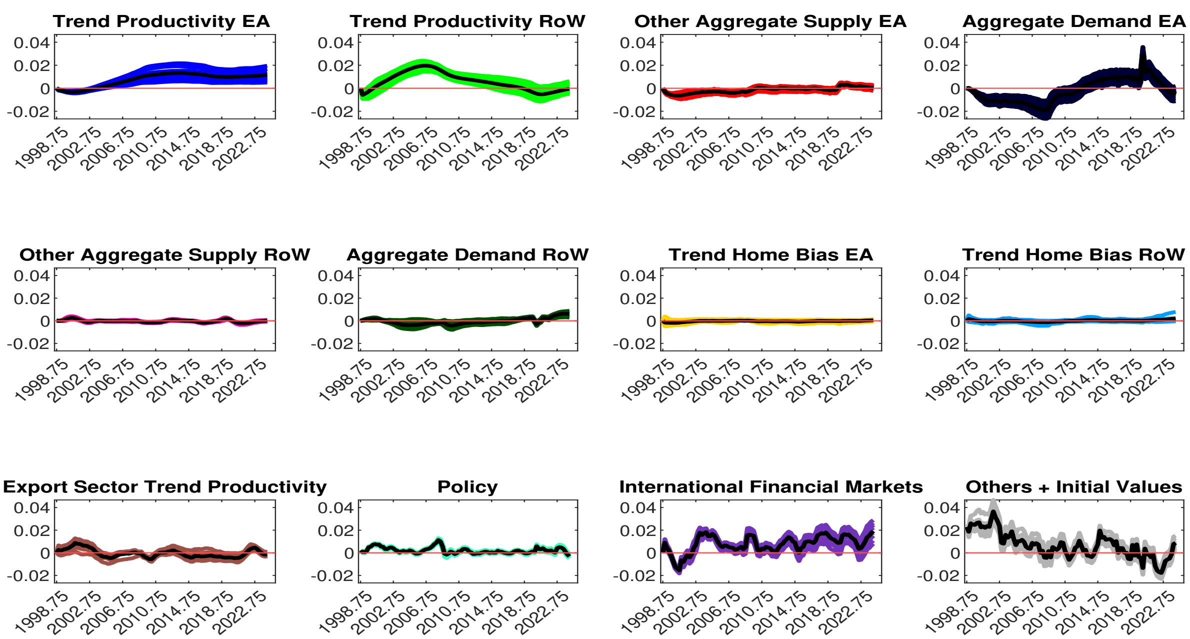


Figure C.4: Endogenous trend growth (risk premium): historical decomposition of EA TB/GDP with parameter uncertainty bands

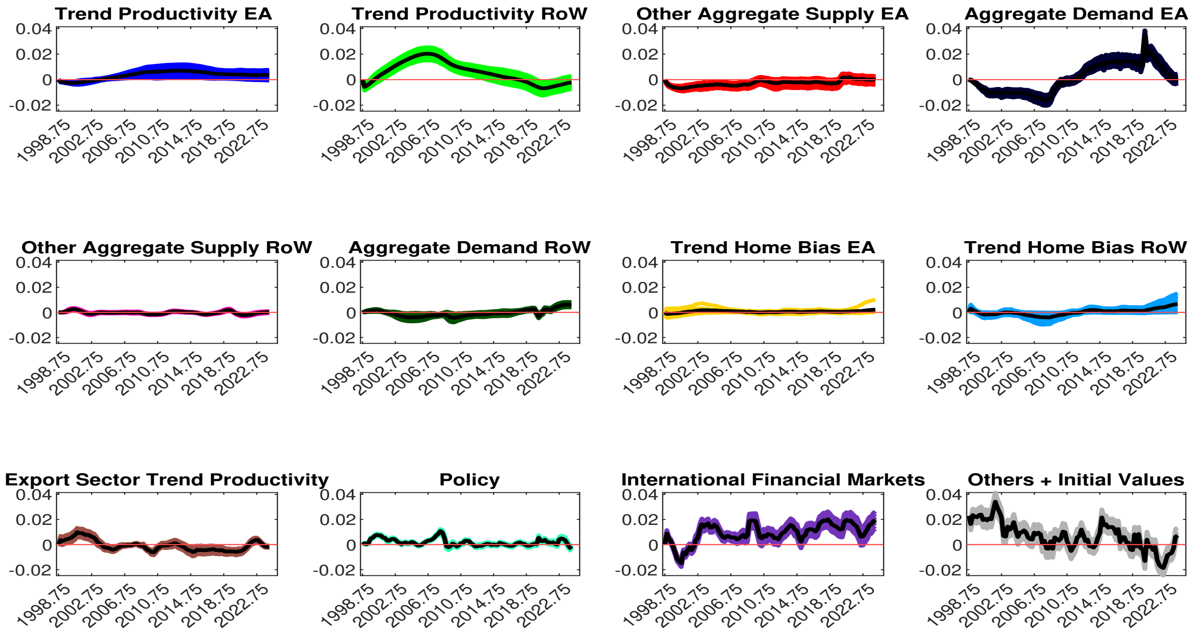


Figure C.5: Endogenous trend growth (fiscal stance): historical decomposition of EA TB/GDP with parameter uncertainty bands

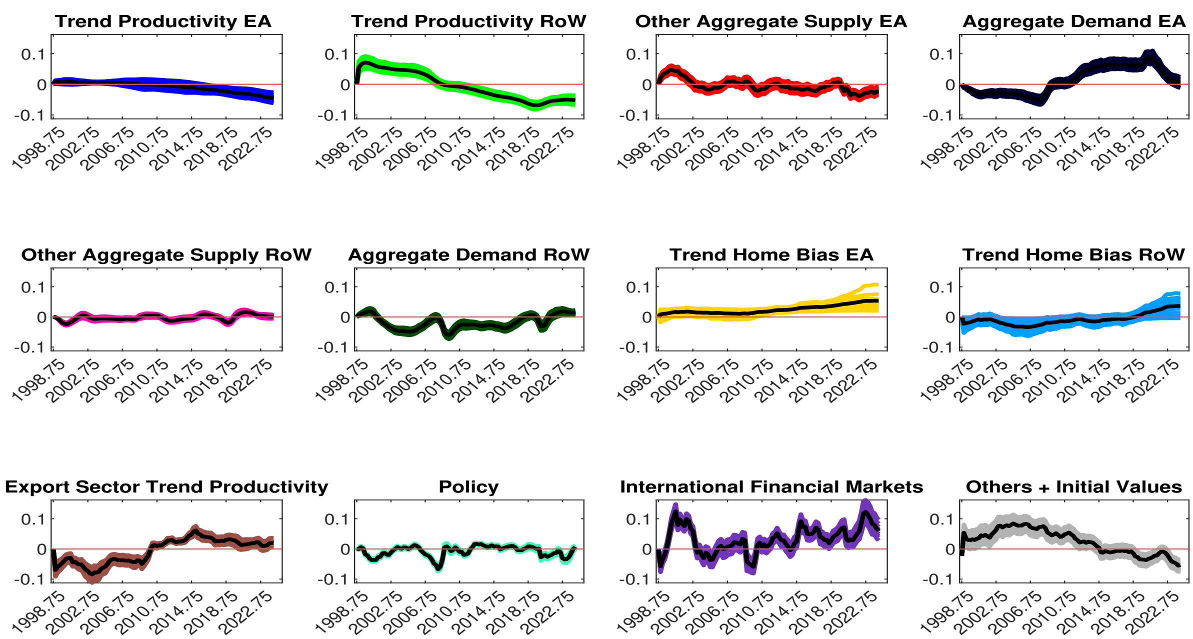


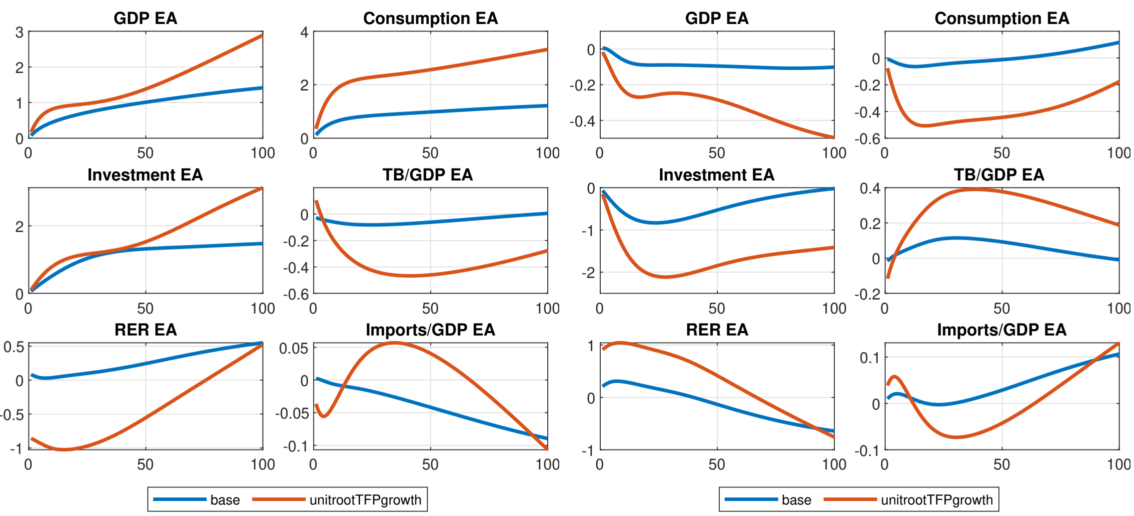
Figure C.6: Endogenous trend growth (fiscal stance): historical decomposition of EA Real Exchange Rate with parameter uncertainty bands

C.2 Near-permanent productivity *growth* differentials

The baseline model assumes identical long-run productivity growth rates across regions to preserve a balanced growth path and allow for standard detrending. Permanent cross-country differences in trend growth would eliminate a balanced growth path and require additional modelling assumptions.

At the same time, our estimates point to very persistent productivity *growth* differences. To assess robustness, we therefore consider a near-permanent productivity growth shock by setting the autoregressive coefficient to $\rho^{AY} = 0.999$. This exercise does not relax the maintained assumption of common long-run productivity growth rates across regions. Rather, it asks whether our main results are sensitive to making productivity-growth shocks arbitrarily persistent within the balanced-growth framework.

Figure C.7 reports impulse responses under this calibration. Higher persistence amplifies and prolongs trade balance surpluses through stronger intertemporal absorption, while leaving the qualitative dynamics emphasized in the paper unchanged.



(a) IRF to EA prod. growth shock

(b) IRF to RoW prod. growth shock

Figure C.7: Robustness to near-permanent growth differences

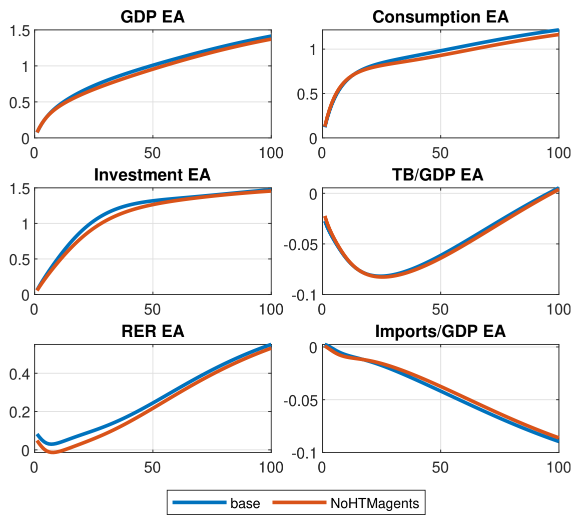
C.3 Nominal rigidities and household heterogeneity

As is common in richer estimated open-economy models, the baseline specification features nominal rigidities and household heterogeneity (e.g. Galí et al. (2007)), allowing domestic demand, monetary policy, and fiscal shocks to affect real activity and external balances at business-cycle frequencies. This section assesses the quantitative role of these features.

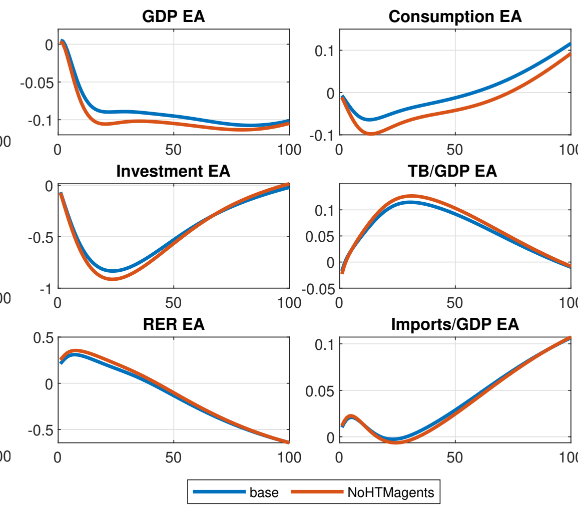
Figures C.8–C.9 report impulse responses under alternative specifications in which model features are modified one at a time. Nominal rigidities are removed by setting price and wage adjustment costs to zero ($\gamma^P = \gamma^W = 0$), while household heterogeneity is eliminated by setting the share of Ricardian households to one ($\omega^s = 1$). Each panel shows two lines corresponding to the baseline (estimated) specification and the alternative model.

The transmission of non-stationary productivity growth shocks is largely unchanged, indicating that nominal rigidities are not central for the medium- and long-run dynamics emphasized in the paper. By contrast, the short-run impact of domestic demand shocks (Figure C.9) is reduced in the absence of nominal rigidities.

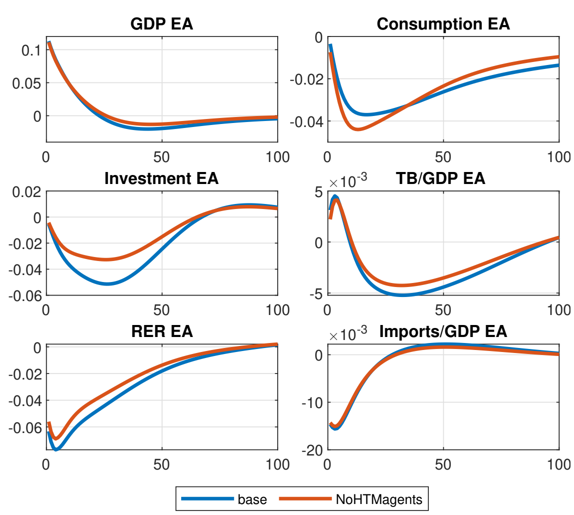
Removing hand-to-mouth households leaves the response to productivity growth shocks essentially unaffected, while differences arise mainly for fiscal shocks. Government spending shocks have a somewhat larger effect on domestic activity and net exports in the presence of hand-to-mouth households, whereas transfer shocks generate non-negligible responses only in the two-agent economy (Figure C.8).



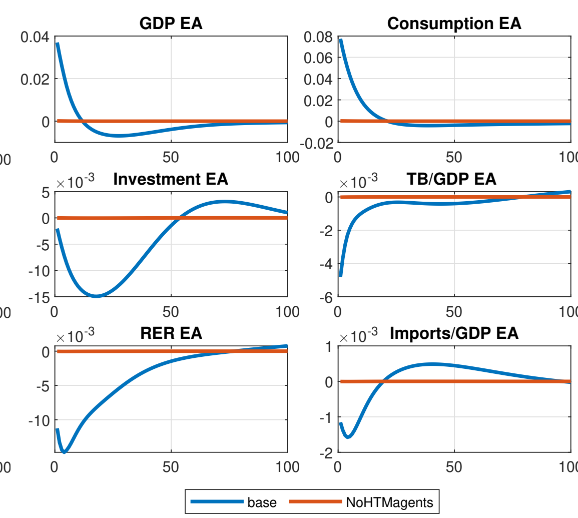
(a) IRF to EA prod. growth shock



(b) IRF to RoW prod. growth shock

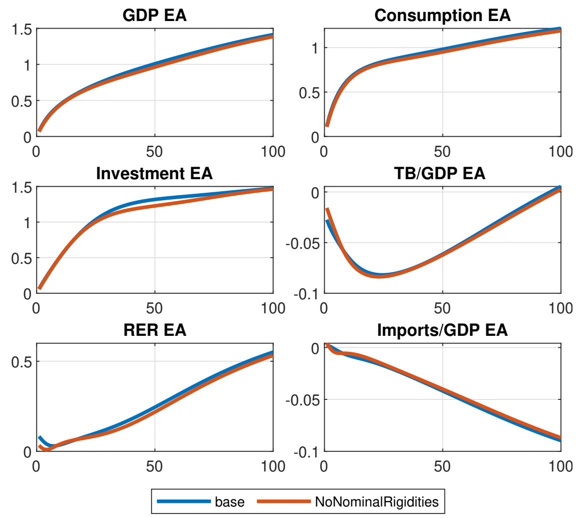


(c) IRF to EA gov. spending shock

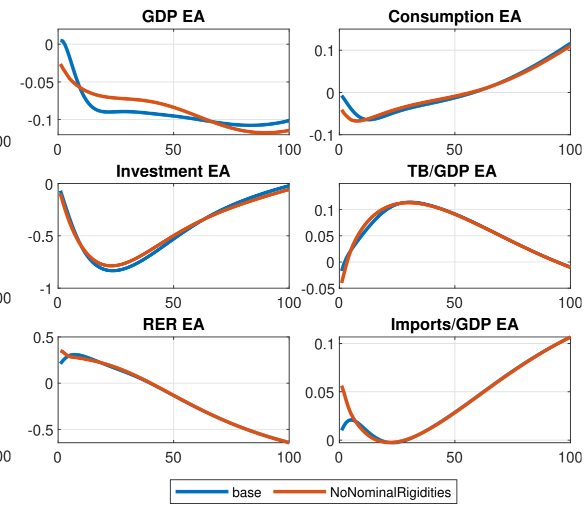


(d) IRF to EA transfer shock

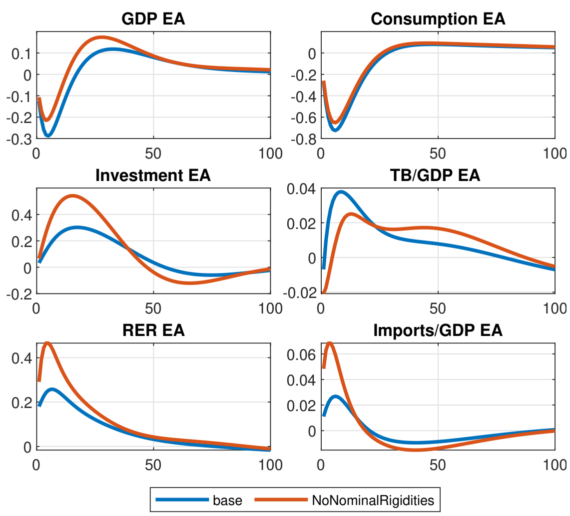
Figure C.8: Robustness to hand-to-mouth households



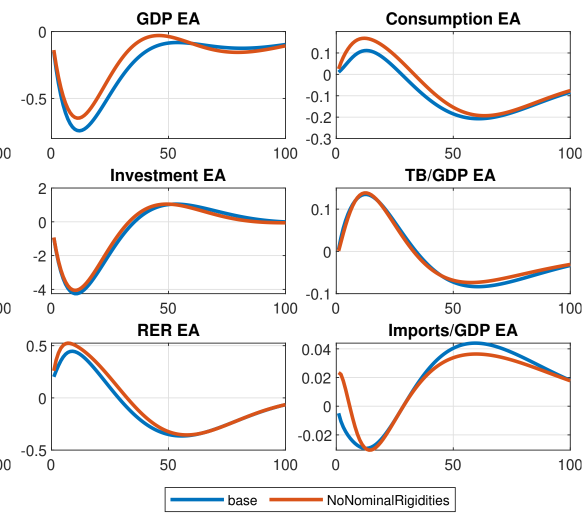
(a) IRF to EA prod. growth shock



(b) IRF to RoW prod. growth shock



(c) IRF to EA preference shock



(d) IRF to EA risk premium shock

Figure C.9: Robustness to nominal rigidities

C.4 Export price setting, pass-through, and the terms of trade

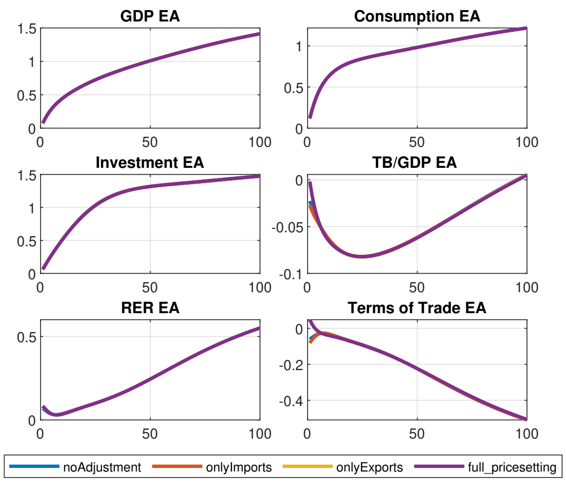
We also analyze the role of export price adjustment costs, governed by γ^{PX}. When $\gamma^{PX} = 0$, export prices are set in producer currency and adjust immediately to changes in marginal costs and the nominal exchange rate. Exchange-rate pass-through is therefore complete in the short run, so relative export prices move one-for-one with the exchange rate and the terms of trade closely track movements in the real exchange rate.

When $\gamma^{PX} > 0$, exporters face costs of adjusting foreign-currency export prices and therefore smooth price changes intertemporally. Exchange-rate fluctuations are then absorbed partly through variations in markups rather than fully through relative prices, implying incomplete pass-through in the short run. As a result, the real exchange rate responds immediately to nominal exchange-rate movements, while the terms of trade adjust more gradually as export prices converge to their new optimal level.

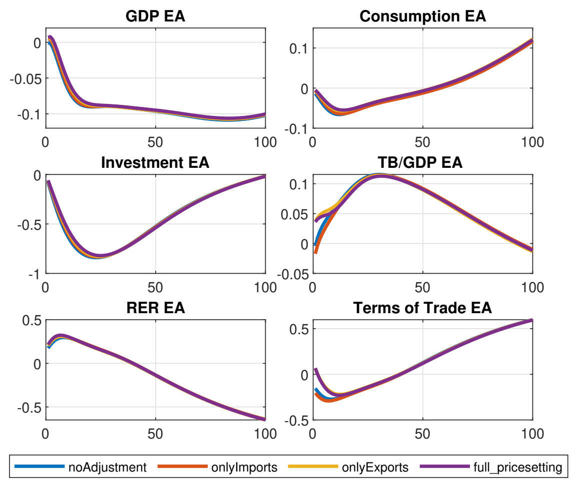
This mechanism is illustrated in Figure C.10, which shows impulse responses to key shocks under these alternative price-setting assumptions: fully flexible import and export prices (blue), quadratic adjustment costs for importing firms (baseline; red), pricing-to-market by exporters (yellow), and nominal rigidities for both importing and exporting firms (purple). Although these specifications yield different short-run dynamics for the terms of trade and the trade balance, the medium- and long-run responses are nearly identical, indicating that our main results are robust to alternative forms of price-setting behavior of import/export firms.

Under pricing-to-market, the real exchange rate adjusts on impact, whereas the terms of trade exhibit a more gradual response. For shocks originating in export and import pricing, the terms of trade can initially move in the opposite direction to the real exchange rate, reflecting short-run adjustments in markups and incomplete pass-through.

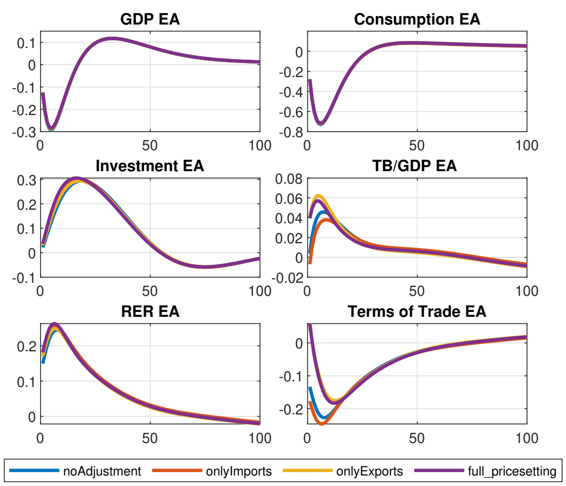
Quantitatively, although pricing-to-market introduces a larger short-run wedge between terms of trade and real exchange rate dynamics, the broader macroeconomic transmission remains largely unchanged.



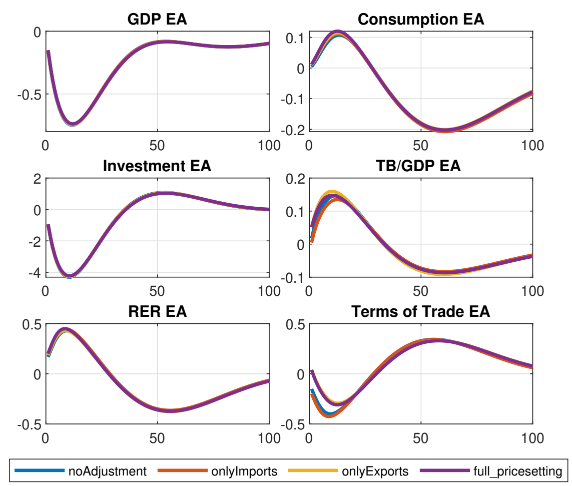
(a) IRF to EA prod. growth shock



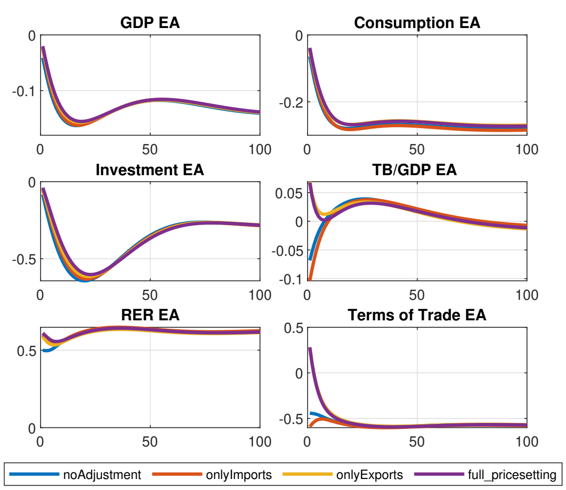
(b) IRF to RoW prod. growth shock



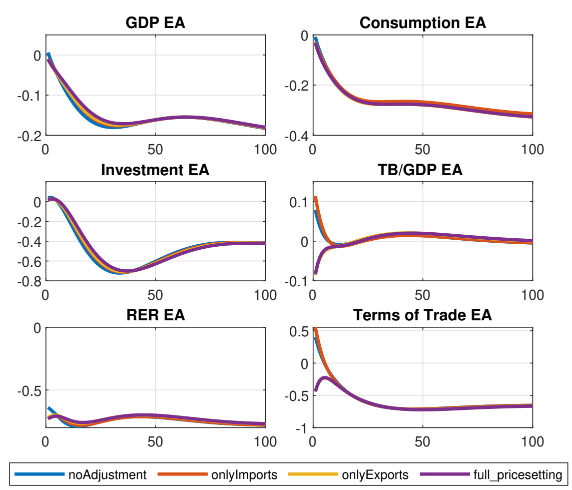
(c) IRF to EA preference shock



(d) IRF to EA risk premium shock



(e) IRF to EA trend home bias shock



(f) IRF to RoW export trend productivity

Note: EA terms of trade is defined as the ratio of export to import prices, both expressed in the EA currency. The baseline estimated model from the main text assumes only Import price adjustment costs (red line in the figures above).

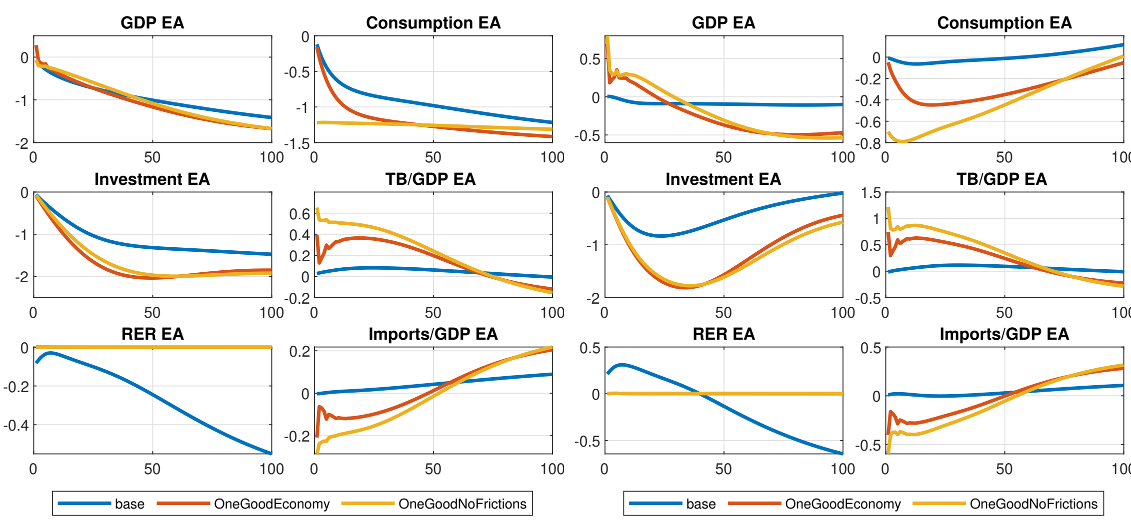
Figure C.10: Robustness to nominal adjustment of import and export prices.

C.5 Imperfect trade substitution

We also investigate the mechanism through which a trend productivity growth rate shock affects the TB. In the estimated model, such shocks generate a highly persistent TB response. This contrasts with Hoffmann et al. (2019), whose two-country, one-good DSGE model predicts a large front-loaded TB adjustment to a productivity trend growth rate shock, under full information, where agents fully observe the persistence of shocks.

We show that the gradual adjustment in our model stems from two features: imperfect substitution between domestic and foreign traded goods, and real adjustment frictions. Figure C.11 reports impulse responses to a negative EA productivity trend growth rate shock, across three model variants: (i) the estimated baseline model (blue lines); (ii) a one-traded-good version with perfect substitution between domestic and foreign tradables ($\sigma_z \approx \infty$, all other parameters kept at baseline values, red lines); and (iii) a version of (ii), without real adjustment frictions (yellow) lines; removing consumption habits, "hand-to-mouth" households, thereby approximating the environment in Hoffmann et al. (2019).

Under perfect substitution (red lines), the terms of trade, and hence the RER, are effectively constant. This amplifies the response of EA absorption and EA TB to the trend productivity growth shock, relative to the baseline model. Removing real frictions further amplifies the TB response (yellow lines). Hoffmann et al. (2019) argue for a theoretical setting with imperfect information—where agents gradually learn about the persistence of shocks—as a mechanism to generate smoother TB responses to trend growth shocks (see also Engel and Rogers (2006)). By contrast, our model assumes full information, yet still generates gradual TB dynamics. Figure C.11 shows that imperfect substitution and real frictions are sufficient to explain this result.



(a) IRF to (negative) EA prod. growth shock (b) IRF to RoW prod. growth shock

Figure C.11: Sensitivity analysis of trade elasticity

D Model description

The model shares many standard elements with Albonico et al. (2019), and we also refer to the model description contained therein.

Model details compared to main text. This appendix provides a more detailed representation of the model described in the main text. The underlying economic structure is identical. In particular, some variables that are decomposed into subcomponents (as mentioned in the main text) are represented here using more detailed building blocks (e.g., disaggregated consumption or investment bundles). These definitions are fully consistent with the underlying economic mechanisms described in the main text.

Notation. Notation follows the main text as closely as possible. Variables referring to the euro area (EA) and the rest of the world (RoW) are indexed explicitly using subscripts. Where additional notation is introduced (e.g., Z_t, MZ_t), it corresponds to objects already defined in the main text: specifically, Z_t denotes domestically produced intermediate inputs (corresponding to Y_t in the main text), while MZ_t denotes imported intermediate inputs.

D.1 Production

D.1.1 Overview

Figure D.1 illustrates the production side of the model, highlighting the nesting of intermediate inputs and their aggregation into final goods.

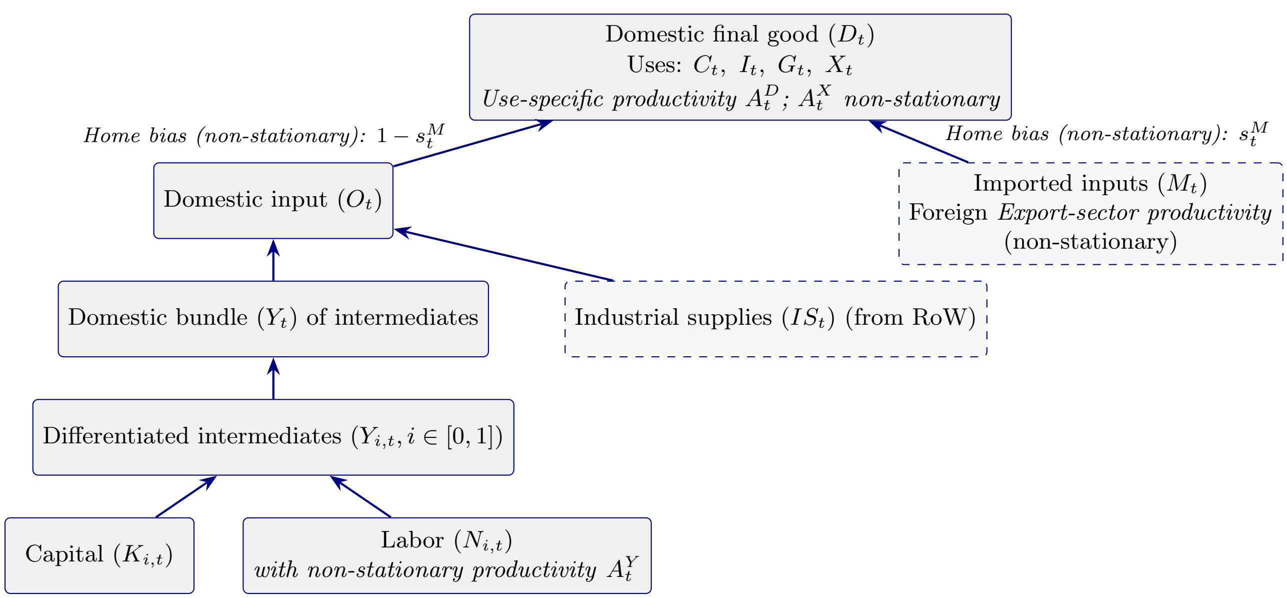


Figure D.1: Production structure

D.1.2 Differentiated intermediate goods

Each firm $i \in [0,1]$ produces a variety of the domestic good, which is an imperfect substitute for varieties produced by other firms. Firms are monopolistically competitive and face a downward-sloping demand function for their individual varieties.

Differentiated goods are produced using capital $K_{i,t-1}$ and labour $N_{i,t}$, combined in a Cobb–Douglas production function:

$$Y_{i,t} = \left(A_t^Y (N_{i,t} - FN)\right)^{\alpha} (cu_{i,t} K_{i,t-1})^{1-\alpha} \left(K_{i,t-1}^G\right)^{1-\alpha_G} - \left(A_t^Y\right)^{\frac{\alpha}{\alpha+\alpha_G-1}} \Phi, \tag{D.1}$$

where α is the steady-state labour share, A_t^Y is labour-augmenting productivity (common across firms), $cu_{i,t}$ is firm-specific capital utilisation, and Φ captures fixed costs in production. $1-\alpha_G$ denotes the output elasticity with respect to public capital. A_t^Y follows a non-stationary process defined in the main text (equations 1–3).

Total hours paid are given by:

$$N_{i,t}^{\text{paid}} = Empl_{i,t} \cdot Hpere_{i,t},$$

and firms can adjust at both the extensive and intensive labour margins, subject to adjustment costs. During the COVID-19 pandemic, we introduce a labour hoarding shock ε_t^{LU} to capture the wedge between paid hours and effective hours worked:

$$\frac{N_{i,t}}{N_{i,t}^{\text{paid}}} = 1 - \varepsilon_t^{LU}. \tag{D.2}$$

The firm maximises the real market value of the firm, equal to the discounted stream of real dividends:

$$\max_{P_{i,t},\, N_{i,t},\, I_{i,t},\, cu_{i,t},\, K_{i,t}} E_t \sum_{s=t}^{\infty} \Lambda_{t,s}^E \frac{div_{i,s}}{P_s}, \tag{D.3}$$

where $\Lambda_{t,s}^E \equiv \prod_{u=t}^{s-1} \Lambda_{u,u+1}^E$ is the equity-market stochastic discount factor.

We assume that the equity-market stochastic discount factor differs from the household stochastic discount factor by a wedge that captures time-varying convenience yields on equity:

$$\Lambda_{t,t+1}^E = \Lambda_{t,t+1}^S \frac{1}{1 + \alpha^{S0} + \varepsilon_t^S}. \tag{D.4}$$

Equivalently, the equity-market stochastic discount factor satisfies

$$1 = E_t \left[\Lambda_{t,t+1}^E (1 + i_{t+1}^s)\right], \qquad 1 + i_{t+1}^s = \frac{P_{t+1}^S + div_{t+1}^S}{P_t^S}.$$

The firm's real dividend is:

$$\frac{div_{i,t}}{P_t} = (1-\tau^K) \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} N_{i,t}^{\text{paid}}\right) + \tau^K \delta \frac{P_t^I}{P_t} K_{i,t-1} - \frac{P_t^I}{P_t} I_{i,t} + \tau^{LU} (N_{i,t}^{paid} - N_{i,t}) - \Gamma_{i,t}, \tag{D.5}$$

where τ^K is the corporate tax rate, δ is the depreciation rate and τ^{LU} is the degree of labour hoarding subsidy to firms by the government.

Firms face quadratic adjustment costs:

$$\Gamma_{i,t} = \Gamma^P_{i,t} + \Gamma^E_{i,t} + \Gamma^H_{i,t} + \Gamma^I_{i,t} + \Gamma^{cu}_{i,t}, \tag{D.6}$$

defined as:

$$\Gamma^P_{i,t} = \sigma^y \frac{\gamma^P}{2} Y_t \left(\frac{P_{i,t}}{P_{i,t-1}} - \exp(\bar{\pi}) \right)^2, \tag{D.7}$$

$$\Gamma^E_{i,t} = \frac{\gamma^E}{2} \left(\frac{Empl_{i,t}}{Empl_{i,t-1}} - \exp(g^{pop}) \right)^2, \tag{D.8}$$

$$\Gamma^H_{i,t} = \frac{W_t}{P_t} Empl_{i,t} Hpere^{trend}_t \left[\gamma^{H,1}(Hpere_{i,t} - 1) + \frac{\gamma^{H,2}}{2}(Hpere_{i,t} - 1)^2 \right], \tag{D.9}$$

$$\Gamma^I_{i,t} = \frac{P^I_t}{P_t} \left[\frac{\gamma^{I,2}}{2} \frac{(I_{i,t} - I_{i,t-1}\exp(g^Y))^2}{K_{t-1}} \right], \tag{D.10}$$

$$\Gamma^{cu}_{i,t} = \frac{P^I_t}{P_t} K_{i,t-1} \left[\gamma^{cu,1}(cu_{i,t} - 1) + \frac{\gamma^{cu,2}}{2}(cu_{i,t} - 1)^2 \right]. \tag{D.11}$$

Here, the denominator K_{t-1} in $\Gamma^I_{i,t}$ denotes aggregate capital, so that investment adjustment costs are scaled by the size of the economy.

Trend terms g^{pop} and g^Y are, respectively, the trends in population and GDP. δ adjusts depreciation to remove trend-path adjustment costs:[1]

The first-order conditions (FOCs) with respect to labour inputs, capital, investment, and utilisation are:

$$\alpha \frac{\mu^y_t P_t Y_t}{W_t Empl_t Hpere^{\text{trend}}_t} = Hpere_t \left[\gamma^{H,1} + \gamma^{H,2}(Hpere_t - 1 + \varepsilon^{Hpere}_t) \right], \tag{D.12}$$

$$(1 - \tau^K)\frac{W_t}{P_t} = \alpha(\mu^y_t - \varepsilon^{ND}_t)\frac{Y_t}{Empl_t - FN} - \frac{\partial \Gamma^E_t}{\partial Empl_t} + E_t \left[\frac{1 + \pi_{t+1}}{1 + i^s_{t+1}} \frac{\partial \Gamma^E_{t+1}}{\partial Empl_t} \right], \tag{D.13}$$

$$Q_t = E_t \left[\frac{1 + \pi_{t+1}}{1 + i^s_{t+1}} \frac{P^I_{t+1}}{P_{t+1}} \frac{P_t}{P^I_t} \left(\tau^K \delta - \frac{\partial \Gamma^{cu}_t}{\partial K_{t-1}} + Q_{t+1}(1 - \delta) + (1 - \alpha)\mu^y_{t+1} \frac{P_{t+1}}{P^I_{t+1}} \frac{Y_{t+1}}{K_t} \right) \right], \tag{D.14}$$

$$\begin{aligned} Q_t = {} & 1 + \gamma^{I,2} \frac{(I_t - I_{t-1}\exp(g^Y))}{K_{t-1}} \\ & - E_t \left[\frac{1 + \pi_{t+1}}{1 + i^s_{t+1}} \frac{P^I_{t+1}}{P_{t+1}} \frac{P_t}{P^I_t} \exp(g^Y)\gamma^{I,2} \frac{(I_{t+1} - I_t \exp(g^Y))}{K_t} \right], \end{aligned} \tag{D.15}$$

$$\mu^y_t (1 - \alpha) \frac{Y_t}{cu_t} \frac{P_t}{P^I_t} = K_{t-1} \left[\gamma^{cu,1} + \gamma^{cu,2}(cu_t - 1) \right], \tag{D.16}$$

where $Q_t = \mu^y_t \cdot \frac{P_t}{P^I_t}$.

[1] We specify $\delta^K_t = \exp(g^Y) - (1 - \bar{\delta})$ so that $\frac{I}{K} - \delta^K = 0$ on trend. For ease of notation, we use δ directly.

In a symmetric equilibrium ($P_{i,t} = P_t$), the FOC for $P_{i,t}$ yields a New Keynesian Phillips Curve:

$$\begin{aligned}\mu_t^y \sigma^y = (1-\tau^K)(\sigma^y - 1) + \sigma^y \gamma^P \frac{P_t}{P_{t-1}}(\pi_t - \bar{\pi}) \\ - \sigma^y \gamma^P \left[\frac{1+\pi_{t+1}}{1+i_{t+1}^s} \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - \bar{\pi}) \right] + \sigma^y u_t^\mu,\end{aligned} \tag{D.17}$$

where u_t^μ is a white noise markup innovation.

D.1.3 Intermediates bundles

Intermediate inputs O_t are a CES aggregate of domestic intermediates Z_t and imported intermediates MZ_t:

$$O_t = \left[(1 - s_t^{MZ})^{1/\sigma^z} Z_t^{(\sigma^z - 1)/\sigma^z} + (s_t^{MZ})^{1/\sigma^z} MZ_t^{(\sigma^z-1)/\sigma^z} \right]^{\sigma^z/(\sigma^z - 1)}, \tag{D.18}$$

where s_t^{MZ} is the stochastic share of imported intermediates, and σ^z is the elasticity of substitution.

The corresponding demand functions are:

$$Z_t = (1 - s_t^{MZ}) \left(\frac{P_t^Z}{P_t^O} \right)^{-\sigma^z} O_t, \tag{D.19}$$

$$MZ_t = s_t^{MZ} \left(\frac{P_t^{MZ}}{P_t^O} \right)^{-\sigma^z} O_t. \tag{D.20}$$

The CES price index for O_t is

$$P_t^O = \left[(1 - s_t^{MZ})(P_t^Z)^{1-\sigma^z} + s_t^{MZ} (P_t^{MZ})^{1-\sigma^z} \right]^{1/(1-\sigma^z)}. \tag{D.21}$$

D.1.4 Final good bundles

Final demand comprises household consumption (starting with the non-energy component), government consumption and investment, private investment, and exports. Each component $\mathcal{D}_t \in \{CFG_t, G_t, I_t, IG_t, X_t\}$ is produced by a perfectly competitive firm combining domestically produced intermediates $O_t^{\mathcal{D}}$ with imported goods $M_t^{\mathcal{D}}$ through a CES aggregator:

$$\mathcal{D}_t = A_t^{p,\mathcal{D}} \left[\left(1 - s_t^{M,\mathcal{D}}\right)^{\frac{1}{\sigma^z}} (O_t^{\mathcal{D}})^{\frac{\sigma^z - 1}{\sigma^z}} + \left(s_t^{M,\mathcal{D}}\right)^{\frac{1}{\sigma^z}} (M_t^{\mathcal{D}})^{\frac{\sigma^z-1}{\sigma^z}} \right]^{\frac{\sigma^z}{\sigma^z - 1}}, \tag{D.22}$$

where $A_t^{p,\mathcal{D}}$ is a sector-specific productivity shock and $s_t^{M,\mathcal{D}}$ is the (stochastic) import share.[2]

Profit maximisation yields the input demand functions:

$$O_t^{\mathcal{D}} = \left(A_t^{p,\mathcal{D}}\right)^{\sigma^z - 1} \left(1 - s_t^{M,\mathcal{D}}\right) \left(\frac{P_t^{\mathcal{D}}}{P_t^O} \right)^{\sigma^z} \mathcal{D}_t, \qquad M_t^{\mathcal{D}} = \left(A_t^{p,\mathcal{D}}\right)^{\sigma^z - 1} s_t^{M,\mathcal{D}} \left(\frac{P_t^{\mathcal{D}}}{P_t^M} \right)^{\sigma^z} \mathcal{D}_t, \tag{D.23}$$

with the corresponding price index:

$$P_t^{\mathcal{D}} = \left(A_t^{p,\mathcal{D}}\right)^{-1} \left[(1 - s_t^{M,\mathcal{D}})(P_t^O)^{1-\sigma^z} + s_t^{M,\mathcal{D}} (P_t^M)^{1-\sigma^z} \right]^{\frac{1}{1-\sigma^z}}. \tag{D.24}$$

[2] The shock applies to all $\mathcal{D}$ except CFG, where it enters at the level of the final consumption aggregator in (D.25).

D.1.5 Final consumption bundles

Final consumption C_t combines non-energy final consumption goods CFG_t with imported energy commodities E^c_t through a CES technology:

$$C_t = A_t^{p,\mathcal{C}} \left[\left(s_t^{CFG}\right)^{\frac{1}{\sigma^{CE}}} CFG_t^{\frac{\sigma^{CE}-1}{\sigma^{CE}}} + \left(1 - s_t^{CFG}\right)^{\frac{1}{\sigma^{CE}}} \left(E_t^c\right)^{\frac{\sigma^{CE}-1}{\sigma^{CE}}} \right]^{\frac{\sigma^{CE}}{\sigma^{CE}-1}}, \tag{D.25}$$

where $A_t^{p,\mathcal{C}}$ is a sector-specific productivity shock, s_t^{CFG} is the share of non-energy goods in the consumption bundle,[3] and σ^{CE} is the elasticity of substitution between the two inputs.

Cost minimisation by the consumption-goods firm, subject to (D.25), yields the demand functions:

$$CFG_t = s_t^{CFG} \left(\frac{P_t^{CFG}}{P_t^c}\right)^{-\sigma^{CE}} C_t, \qquad E_t^c = (1 - s_t^{CFG}) \left(\frac{P_t^E}{P_t^c}\right)^{-\sigma^{CE}} C_t, \tag{D.26}$$

with the corresponding price index:

$$P_t^c = \left(A_t^{p,\mathcal{C}}\right)^{-1} \left[s_t^{CFG} \left(P_t^{CFG}\right)^{1-\sigma^{CE}} + \left(1 - s_t^{CFG}\right) \left(P_t^E\right)^{1-\sigma^{CE}} \right]^{\frac{1}{1-\sigma^{CE}}}. \tag{D.27}$$

The non-energy component CFG_t is produced as in Section D.1.4, while E_t^c represents the direct energy input to consumption.

D.2 Delayed substitution

We follow Auclert et al. (2024) in modeling delayed substitution. We first log-linearize the standard CES demand functions and allow for forward-looking dynamics. Finally, we introduce inertia, capturing delayed adjustment patterns over time.

The delayed-substitution friction applies to the choice between *domestic and imported intermediate inputs* within $\mathcal{D}_t$, and therefore affects only the elasticity σ^z. Conditional on either bundle, firms can freely reallocate across varieties; there is no sluggish adjustment within domestic or imported intermediates.

We define the domestic and imported target bundles, $\mathcal{XOD}_t$ and $\mathcal{XMD}_t$, as the ratios of, respectively, $O_t^{\mathcal{D}}$ and $M_t^{\mathcal{D}}$, to the demand component $\mathcal{D}_t$. Taking the log-linear approximation of (D.23) we obtain:

$$\log\left(\mathcal{XOD}_t\right) - \log(1 - s_t^{M,\mathcal{D}}) = (\sigma^z - 1)\log(A_t^{p,\mathcal{D}}) + \sigma^z \log\left(\frac{P_t^{\mathcal{D}}}{P_t^O}\right) \tag{D.28}$$

$$\log\left(\mathcal{XMD}_t\right) - \log(s_t^{M,\mathcal{D}}) = (\sigma^z - 1)\log(A_t^{p,\mathcal{D}}) + \sigma^z \log\left(\frac{P_t^{\mathcal{D}}}{P_t^M}\right) \tag{D.29}$$

[3] s_t^{CFG} is time-varying, with the same disturbance as s_t^{EP}. The two shares are linked through the relative use of energy commodities in production versus consumption: $s_t^{CFG} = 1 - (s_t^E - s_t^{EP})Z_t/C_t$, where s_t^E is the share of energy commodities in total output.

Domestic and imported bundles are reset by consumers based on their perceptions of current and future relative prices:

$$\log(\mathcal{XOD}_t) - \log(1 - s_t^{M,\mathcal{D}}) = (1 - \tilde{\beta}_t \rho^z)\Big[(\sigma^z - 1)\log(A_t^{p,\mathcal{D}}) + \sigma^z \log\Big(\frac{P_t^{\mathcal{D}}}{P_t^O}\Big)\Big] + \tilde{\beta}_t \rho^z \Big[\log(\mathcal{XOD}_{t+1}) - \log(1 - s_{t+1}^{M,\mathcal{D}})\Big] \tag{D.30}$$

$$\log(\mathcal{XMD}_t) - \log(s_t^{M,\mathcal{D}}) = (1 - \tilde{\beta}_t \rho^z)\Big[(\sigma^z - 1)\log(A_t^{p,\mathcal{D}}) + \sigma^z \log\Big(\frac{P_t^{\mathcal{D}}}{P_t^M}\Big)\Big] + \tilde{\beta}_t \rho^z \Big[\log(\mathcal{XMD}_{t+1}) - \log(s_{t+1}^{M,\mathcal{D}})\Big] \tag{D.31}$$

where ρ^z governs the degree of inertia. The demand for the domestic and imported bundles, in turn, is assumed to evolve sluggishly:

$$\log\Big(\frac{O_t^{\mathcal{D}}}{\mathcal{D}_t}\Big) = (1 - \rho^z)\log\Big(\mathcal{XOD}_t\Big) + \rho^z \log\Big(\frac{O_{t-1}^{\mathcal{D}}}{\mathcal{D}_{t-1}}\Big) \tag{D.32}$$

$$\log\Big(\frac{M_t^{\mathcal{D}}}{\mathcal{D}_t}\Big) = (1 - \rho^z)\log\Big(\mathcal{XMD}_t\Big) + \rho^z \log\Big(\frac{M_{t-1}^{\mathcal{D}}}{\mathcal{D}_{t-1}}\Big). \tag{D.33}$$

D.3 Importers

Perfectly competitive importer bundlers combine differentiated imports into a single import good

$$M_t = \Big(\int_0^1 M_{it}^{\frac{\sigma^M - 1}{\sigma^M}} di\Big)^{\frac{\sigma^M}{\sigma^M - 1}}, \tag{D.34}$$

where M_{it} denotes intermediate variety $i \in [0, 1]$. $\sigma^M > 0$ is the elasticity of substitution between the varieties M_{it}. Profit maximisation gives rise to a standard demand function for different import varieties:

$$M_{it} = \Big(\frac{P_{it}^M}{P_t^M}\Big)^{-\sigma^M} M_t \tag{D.35}$$

Monopolistically competitive importers choose their individual price P_{it}^M to maximise expected discounted dividends,

$$\max_{\{P_{it}^M\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^E div_{i,t}^M, \tag{D.36}$$

where

$$div_{i,t}^M = \frac{P_{it}^M}{P_t} M_{it} - \mathcal{E}_t \frac{P_t^{X,*}}{P_t} M_{it} - \Gamma_{i,t}^{PM} \tag{D.37}$$

with M_{it} denoting demand as specified in (D.35). Dividends consist of sales revenues net of import costs and quadratic price adjustment costs

$$\Gamma_{i,t}^{PM} = \frac{(\sigma^M - 1)\gamma^{PM}}{2} M_t \left[\frac{P_{i,t}^M}{P_{i,t-1}^M \tilde{\pi}_{t-1}^M} - 1\right]^2, \tag{D.38}$$

where $\tilde{\pi}_t^M \equiv (\pi_t^M)^{1-sfpm}(\bar{\pi})^{sfpm}$ such that a share $1 - sfpm$ of firms index to lagged import inflation, while the remaining share $sfpm$ index to steady-state inflation.

In a symmetric equilibrium, and with σ^M sufficiently large, the steady-state markup is close to unity, yielding the import Phillips curve[4]:

$$\frac{\pi_t^M}{\tilde{\pi}_{t-1}^M}\Big(\frac{\pi_t^M}{\tilde{\pi}_{t-1}^M} - 1\Big) = E_t\Big[\Lambda_{t,t+1}^E \frac{\pi_{t+1}^M}{\tilde{\pi}_t^M}\frac{M_{t+1}}{M_t}\Big(\frac{\pi_{t+1}^M}{\tilde{\pi}_t^M} - 1\Big)\Big] + \frac{1}{\gamma^{PM}}\Big(\frac{P_t^M}{\mathcal{E}_t P_t^{X,*}} - 1\Big). \tag{D.39}$$

D.4 Commodity importers

Perfectly competitive bundlers aggregate a continuum of differentiated commodity import varieties into a composite good,

$$IS_t = \left(\int_0^1 IS_{i,t}^{\frac{\sigma^{IS}-1}{\sigma^{IS}}}\, di\right)^{\frac{\sigma^{IS}}{\sigma^{IS}-1}}, \tag{D.40}$$

where $\sigma^{IS} > 1$ is the elasticity of substitution across varieties. Cost minimization implies

$$IS_{i,t} = \left(\frac{P_{i,t}^{IS}}{P_t^{IS}}\right)^{-\sigma^{IS}} IS_t. \tag{D.41}$$

Monopolistically competitive commodity importers set $P_{i,t}^{IS}$ to maximize discounted real dividends,

$$\max_{\{P_{i,t}^{IS}\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^S div_{i,t}^{IS}, \tag{D.42}$$

where

$$div_{i,t}^{IS} = \left(\frac{P_{i,t}^{IS}}{P_t} - \frac{P_t^{IS,M}}{P_t} - \tau^{IS}\right) IS_{i,t} - \Gamma_{i,t}^{P,IS}, \tag{D.43}$$

and quadratic price adjustment costs are

$$\Gamma_{i,t}^{P,IS} = \frac{(\sigma^{IS}-1)\gamma^{P,IS}}{2} IS_t \left[\frac{P_{i,t}^{IS}}{P_{i,t-1}^{IS}\,\tilde{\pi}_{t-1}^{IS}} - 1\right]^2, \qquad \tilde{\pi}_t^{IS} \equiv (\pi_t^{IS})^{1-sfpis}(\bar{\pi})^{sfpis}. \tag{D.44}$$

τ^{IS} denotes an excise duty.

In a symmetric equilibrium and assuming σ^{IS} sufficiently large so that $\frac{\sigma^{IS}}{\sigma^{IS}-1} \approx 1$, commodity import inflation satisfies the Rotemberg Phillips curve:

$$\frac{\pi_t^{IS}}{\tilde{\pi}_{t-1}^{IS}}\left(\frac{\pi_t^{IS}}{\tilde{\pi}_{t-1}^{IS}} - 1\right) = E_t\left[\Lambda_{t,t+1}^S \frac{\pi_{t+1}^{IS}}{\tilde{\pi}_t^{IS}}\frac{IS_{t+1}}{IS_t}\left(\frac{\pi_{t+1}^{IS}}{\tilde{\pi}_t^{IS}} - 1\right)\right] + \frac{1}{\gamma^{P,IS}}\left(\frac{P_t^{IS}}{P_t^{IS,M} + \tau^{IS}P_t} - 1\right). \tag{D.45}$$

[4]Formally, we assume: $\frac{\sigma^M}{\sigma^M-1} \approx 1$. We therefore abstract from the small steady-state distortion and focus on the inflation dynamics implied by quadratic adjustment costs.

D.5 Households

Two groups of representative households consume and provide labour to intermediate goods producers. A share ω^s of households are savers (s) who own domestic firms and participate in financial markets (saving and borrowing).

D.5.1 Ricardian households

Ricardian households work, consume, own firms, and receive net transfers from the government. They hold government bonds, a rest-of-world bond, a private risk-free bond (zero net supply), and domestic equity. Financial assets in real terms are

$$\frac{A_{jt}}{P_t^C} = \frac{B_{jt}^{rf}}{P_t^C} + \frac{B_{jt}^g}{P_t^C} + \frac{\mathcal{E}_t B_{jt}^W}{P_t^C} + \frac{P_t^S S_{jt}}{P_t^C},$$

with P_t^C the consumption price net of τ^C (the gross price is $P_t^{C,vat} = (1+\tau^C)P_t^C$).

Net transfers and taxes (excluding profit and capital taxation) are

$$T_t^S = TR_t^S - tax_t^S - \tau^N W_t \left(N_t^S + \varepsilon_t^{tN}\right) - \tau^C P_t^C C_t^S.$$

The nominal budget constraint is

$$\begin{aligned} P_t^C C_{jt}^S + A_{jt} = W_t \left(N_t^S + \varepsilon_t^{tN}\right) + (1+i_{t-1}^g)B_{jt-1}^g + (1+i_{t-1}^W)\mathcal{E}_t B_{jt-1}^W + (1+i_{t-1}^{rf})B_{jt-1}^{rf} \\ + (P_t^S + div_t^S)S_{jt-1} + div_t^{tot} + T_t^S, \end{aligned} \tag{D.46}$$

where all returns and dividends are net of the relevant taxes. Here, div_t^S denotes dividends paid per share on the traded equity, and div_t^{tot} collects dividends/profits from other firms not represented by S_{jt}.

The gross nominal equity return is

$$1 + i_{t+1}^s = \frac{P_{t+1}^S + div_{t+1}^S}{P_t^S}.$$

Preferences (per capita) are

$$u_{jt}^S = \frac{\left(\frac{C_{jt}^S}{POP_t} - \varepsilon_t^{tC} - h\left(\frac{C_{t-1}^S}{POP_{t-1}} - \varepsilon_{t-1}^{tC}\right)\right)^{1-\theta}}{1-\theta} - \omega_t^N \frac{\left(\frac{N_{jt}^S}{POP_t}\right)^{1+\theta^N}}{1+\theta^N} - \left(\frac{C_t^S}{POP_t} - h\frac{C_{t-1}^S}{POP_t}\right)^{-\theta} \frac{U_{jt-1}^A}{P_t^C},$$

with $h \in (0,1)$, COVID-specific consumption shock ε_t^{tC}, and time-varying labour disutility ω_t^N. Since we allow for demographic shocks (active population evolves with region specific paths and shocks), we scale utility by the population size (POP_t) in each period:

Asset disutility is

$$U_{jt-1}^A = (\alpha^{b0} + \varepsilon_{t-1}^g)\frac{B_{jt-1}^g}{POP_t} + (\alpha^{bw0} + \varepsilon_{t-1}^{bw})\mathcal{E}_t \frac{B_{jt-1}^W}{POP_t} + (\alpha^{S0} + \varepsilon_{t-1}^S)\frac{P_{t-1}^S S_{jt-1}}{POP_t}. \tag{D.47}$$

The utility specification introduces asset-specific intercepts $\alpha^{\mathcal{Q}}$ and shocks $\varepsilon_t^{\mathcal{Q}}$ with $\mathcal{Q} \in \{g, bw, S\}$, allowing for a structural interpretation of risk premia and convenience yields (Fisher, 2015).

Households maximise:

$$E_0 \sum_{t=0}^{\infty} \beta^t (POP_t u_{jt}^S - \lambda_t^S (C_{jt}^S + \ldots))$$

FOCs (consumption and asset demands). Let

$$\lambda_t^S = \left(C_t^S - \varepsilon_t^{tC} - h(C_{t-1}^S - \varepsilon_{t-1}^{tC})\right)^{-\theta}, \quad \pi_{t+1}^{C,vat} = \frac{P_{t+1}^{C,vat}}{P_t^{C,vat}}, \quad \Lambda_{t,t+1}^S = \beta \frac{\xi_{t+1}}{\xi_t} \frac{\lambda_{t+1}^S}{\lambda_t^S} \frac{1}{\pi_{t+1}^{C,vat}}, \quad \frac{\xi_{t+1}}{\xi_t} = e^{\varepsilon_t^C}.$$

Then

$$C_{jt}^S: \quad \lambda_t^S = \left(C_t^S - \varepsilon_t^{tC} - h(C_{t-1}^S - \varepsilon_{t-1}^{tC})\right)^{-\theta}, \tag{D.48}$$

$$B_{jt}^{rf}: \quad 1 = E_t\left[\Lambda_{t,t+1}^S (1 + i_t^{rf})\right], \tag{D.49}$$

$$B_{jt}^g: \quad 1 = E_t\left[\Lambda_{t,t+1}^S \left(1 + i_t^g - \varepsilon_t^g - \alpha^{b0}\right)\right], \tag{D.50}$$

$$B_{jt}^W: \quad 1 = E_t\left[\Lambda_{t,t+1}^S \left((1 + i_t^W)\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} - \varepsilon_t^{bw} - \alpha^{bw0}\right)\right], \tag{D.51}$$

$$S_{jt}: \quad 1 = E_t\left[\Lambda_{t,t+1}^S \left(1 + i_{t+1}^s - \varepsilon_t^S - \alpha^{s0}\right)\right] \tag{D.52}$$

We normalise the number of shares to one.

D.5.2 Hand-to-mouth households

The remaining households, with a population share $1 - \omega^s$, are so-called *hand-to-mouth* consumers (denoted c), who face a zero-borrowing constraint and do not participate in asset markets. Each period, these households consume all of their current disposable wage and transfer income. Their consumption is determined by

$$P_t^{C,\text{vat}} C_{j,t}^c = W_t N_{j,t}^{c,\text{paid}} + T_{j,t}^c - P_t^{C,\text{vat}} \left(\varepsilon_t^{tC} - \frac{1}{6}\sum_{i=8}^{13} \varepsilon_{t-i}^{tC}\right). \tag{D.53}$$

Here, $P_t^{C,\text{vat}}$ is the consumption price index including VAT, W_t is the nominal wage, $N_{j,t}^{c,\text{paid}}$ denotes paid hours worked, and $T_{j,t}^c$ collects net transfers, taxes, and social contributions. The terms ε_t^{tC} and ε_{t-i}^{tC} represent COVID-19-related forced savings shocks. During the pandemic, these households accumulate forced savings, which are gradually spent as the pandemic ends.

D.5.3 Aggregation

Economy-wide variables are population-weighted averages; for example, total consumption is $C_t = (1 - \omega^s)C_t^c + \omega^s C_t^s$.

D.5.4 Labour packers

Labour packers aggregate differentiated labour services into a homogeneous input via a CES technology

$$N_t = \left(\int_0^1 N_{lt}^{\frac{\sigma^n - 1}{\sigma^n}} \, dl \right)^{\frac{\sigma^n}{\sigma^n - 1}}, \tag{D.54}$$

where $\sigma^n > 1$ denotes the elasticity of substitution across labour types. Profit maximisation and zero-profit conditions imply the demand for each type-l service:

$$N_{lt} = \left(\frac{W_{lt}}{W_t} \right)^{-\sigma^n} N_t. \tag{D.55}$$

D.5.5 Unions

Each union $l \in [0,1]$ sets its nominal wage W_{lt} to maximise the discounted utility of its representative members:

$$\max_{W_{lt}} U_{l0} = \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_{lt}, \cdot), \tag{D.56}$$

subject to labour demand (D.55) and the household budget constraint:

$$P_t^{C,vat} C_{j,t} + \Gamma_{l,t}^W = (1 - \tau_t^N) W_{lt} N_{lt} + \Xi_t, \tag{D.57}$$

where Ξ_t collects additive terms irrelevant for wage setting. In real consumption units,

$$C_{j,t} + \frac{\Gamma_{l,t}^W}{P_t^{C,vat}} = (1 - \tau_t^N) \frac{W_{lt}}{P_t^{C,vat}} N_{lt} + \frac{\Xi_t}{P_t^{C,vat}}. \tag{D.58}$$

Note that in this Appendix we explicitly include consumption taxes $P_t^{C,vat} = P_t^C (1 + \tau^C)$, which, for brevity, are omitted in the main text.

Nominal wage adjustment is subject to quadratic costs

$$\Gamma_{l,t}^W = \frac{\gamma^W}{2} N_t W_t \left(\frac{W_{lt}}{W_{l,t-1} \tilde{\pi}_{t-1}^W} - 1 \right)^2, \tag{D.59}$$

with γ^W governing wage rigidity. Wage indexation is defined as

$$\tilde{\pi}_t^W = \left(\pi_t^{C,vat} \frac{\bar{\pi}^W}{\bar{\pi}^C} \right)^{1 - sfw} (\bar{\pi}^W)^{sfw}, \tag{D.60}$$

where $\pi_t^{C,vat} = P_t^{C,vat} / P_{t-1}^{C,vat}$ is gross consumption inflation, $\bar{\pi}^W$ denotes steady-state wage inflation, and $sfw \in [0,1]$ is the indexation parameter.

Wage setting condition. Optimality implies the following first-order condition (details omitted):

$$\begin{aligned} 0 = & \, mrs_t - (1 - \tau_t^N) \mu^W \frac{W_t}{P_t^{C,vat}} - \frac{\gamma^W}{\sigma^n} \left[\frac{\pi_t^W}{\tilde{\pi}_t^W} \left(\frac{\pi_t^W}{\tilde{\pi}_t^W} - 1 \right) \right] \frac{W_t}{P_t^{C,vat}} \\ & + \frac{\gamma^W}{\sigma^n} E_t \left[\Lambda_{t,t+1}^{tot} \frac{N_{t+1}}{N_t} \left(\frac{\pi_{t+1}^W}{\tilde{\pi}_{t+1}^W} - 1 \right) \frac{\pi_{t+1}^W}{\tilde{\pi}_{t+1}^W} \frac{W_{t+1}}{P_{t+1}^{C,vat}} \right], \end{aligned} \tag{D.61}$$

where we define $mrs_t \equiv \frac{U_{N,t}}{U_{C,t}}$ (marginal rate of substitution), $\mu^W \equiv \frac{\sigma^n - 1}{\sigma^n}$ (inverse wage markup), and $\Lambda^{tot}_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$ (stochastic discount factor weighted across household groups).

This can be rearranged into the wage Phillips curve:

$$\begin{aligned}\frac{\pi^W_t}{\tilde{\pi}^W_t}\left(\frac{\pi^W_t}{\tilde{\pi}^W_t} - 1\right)\frac{W_t}{P^{C,vat}_t} =& \frac{\sigma^n}{\gamma^W}\left(mrs_t - \mu^W \frac{(1-\tau^N_t)W_t}{P^{C,vat}_t}\right) \\ &+ E_t\left[\Lambda^{tot}_{t,t+1}\frac{N_{t+1}}{N_t}\left(\frac{\pi^W_{t+1}}{\tilde{\pi}^W_{t+1}} - 1\right)\frac{\pi^W_{t+1}}{\tilde{\pi}^W_{t+1}}\frac{W_{t+1}}{P^{C,vat}_{t+1}}\right].\end{aligned} \tag{D.62}$$

D.5.6 Real wage rigidities

Define the effective wage wedge

$$\mu^W_t = \frac{\gamma^W}{\sigma^n}\left(\frac{\pi^W_t}{\tilde{\pi}^W_t}\left(\frac{\pi^W_t}{\tilde{\pi}^W_t} - 1\right)\frac{W_t}{P^{C,vat}_t} - E_t\left[\Lambda^{tot}_{t,t+1}\frac{N_{t+1}}{N_t}\left(\frac{\pi^W_{t+1}}{\tilde{\pi}^W_{t+1}} - 1\right)\frac{\pi^W_{t+1}}{\tilde{\pi}^W_{t+1}}\frac{W_{t+1}}{P^{C,vat}_{t+1}}\right]\right) + \varepsilon^U_t, \tag{D.63}$$

where ε^U_t is the wage markup shock.

Equation (D.62) then reduces to

$$mrs_t - \mu^W_t = \mu^W \frac{(1-\tau^N_t)W_t}{P^{C,vat}_t}. \tag{D.64}$$

To capture real wage rigidities, we allow for gradual adjustment:

$$\left(mrs_t - \mu^W_t\right)^{1-\gamma^{wr}}\left(\mu^W \frac{(1-\tau^N_{t-1})W_{t-1}}{P^{C,vat}_{t-1}}\right)^{\gamma^{wr}} = \mu^W \frac{(1-\tau^N_t)W_t}{P^{C,vat}_t}. \tag{D.65}$$

D.6 Fiscal policy

The government collects taxes on labour, τ^N, corporate profits, τ^K, consumption, τ^C, and lump-sum taxes, tax_t, and issues one-period bonds, B^G_t, to finance government consumption, G_t, public investment, I^G_t, nominal transfers, T_t, and the servicing of the outstanding government debt. The tax on commodity imports from the RoW, τ^{IS}, is fixed. τ^{LU} denotes a labour hoarding subsidy. The government budget constraint is:

$$B^G_t = (1+i^g_{t-1})B^G_{t-1} - R^G_t + P^G_t G_t + P^{IG}_t I^G_t + T_t \tag{D.66}$$

where nominal government revenues R^G_t are defined as:

$$\begin{aligned}R^G_t =& \tau^K(P_tY_t - W_tN^{paid}_t - P^I_t\delta K_{t-1}) + \tau^N W_t N^{paid}_t + \tau^C P^C_t C_t \\ &+ \tau^{IS}P^{IS}_t IS_t + tax_t P_t Y_t + \tau^{LU}W_t\left(N^{paid}_t - N_t\right).\end{aligned} \tag{D.67}$$

The government closes its budget via lump-sum taxes:

$$tax_t = \rho^\tau tax_{t-1} + \eta^d \left(\frac{\Delta B^G_{t-1}}{Y^{pot}_{t-1} P_{t-1}} - \bar{def} \right) + \eta^B \left(\frac{B^G_{t-1}}{Y^{pot}_{t-1} P_{t-1}} - \bar{BG} \right) + \varepsilon^{tax}_t, \tag{D.68}$$

where $\bar{def}$ and $\bar{BG}$ are the targets for the government deficit and the government debt level, with debt rule coefficients η^d and η^B, respectively. ε^{tax}_t is a white noise shock. ρ^τ governs the debt rule persistence. The accumulation equation for government capital is:

$$K^G_t = (1-\delta)K^G_{t-1} + I^G_t, \tag{D.69}$$

where δ is the depreciation rate.

We use the following fiscal rules for government consumption, G_t, investment, I^G_t, and transfers, T_t:

$$\frac{G_t P^G_t}{P_t Y^{pot}_t} = \varepsilon^G_t, \tag{D.70}$$

$$\frac{I^G_t P^{IG}_t}{P_t Y^{pot}_t} = \varepsilon^{IG}_t, \tag{D.71}$$

$$\frac{T_t}{Y^{pot}_t} = \varepsilon^T_t. \tag{D.72}$$

where ε^G_t, ε^{IG}_t, ε^T_t represent shocks to government consumption, investment and transfers, respectively.

D.7 RoW details

The RoW final good. Similar to the EA region, final goods producers combine domestic inputs, $O^{Z,*}_t$, and imported goods, $M^{Z,*}_t$, in a CES production function:

$$Z^*_t = A^{Z,*}_t \left[\left(1 - s^{M,Z,*}_t\right)^{\frac{1}{\sigma^{z,*}}} \left(O^{Z,*}_t\right)^{\frac{\sigma^{z,*}-1}{\sigma^{z,*}}} + \left(s^{M,Z,*}_t\right)^{\frac{1}{\sigma^{z,*}}} \left(M^{Z,*}_t\right)^{\frac{\sigma^{z,*}-1}{\sigma^{z,*}}} \right]^{\frac{\sigma^{z,*}}{\sigma^{z,*}-1}}, \tag{D.73}$$

where $Z^*_t \in \{C^*_t, I^*_t, X^*_t\}$ denotes the demand for final goods by households, private investors, and exporters of final goods, respectively. $A^{Z,*}_t$ denotes a productivity shock in sector Z, and $0 < s^{M,Z,*}_t < 1$ is the stochastic import share associated with the different components of final demand. This is given by $s^{M,Z,*}_t = s^{M,Z,*} \exp(\varepsilon^{M,*}_t)$, where $s^{M,Z,*}$ denotes the steady-state import share of the demand component Z, and $\varepsilon^{M,*}_t$ is a foreign bias shock (shifting home bias preferences). The parameter $\sigma^{z,*} > 0$ is the elasticity of substitution between domestic output and imports in the assembly of the final good. This elasticity is assumed to be common across all final demand components. Price indices in the RoW are defined analogously to the EA, with a more aggregated representation reflecting the simpler import structure.

Output. Perfectly competitive firms produce output (O^*_t) by combining domestic value added (Y^*_t) and imported industrial supplies (IS^*_t) in a CES production function:

$$O^*_t = \left[\left(1 - s^{IS,*}_t\right)^{\frac{1}{\sigma^{o,*}}} \left(Y^*_t\right)^{\frac{\sigma^{o,*}-1}{\sigma^{o,*}}} + \left(s^{IS,*}_t\right)^{\frac{1}{\sigma^{o,*}}} \left(IS^*_t\right)^{\frac{\sigma^{o,*}-1}{\sigma^{o,*}}} \right]^{\frac{\sigma^{o,*}}{\sigma^{o,*}-1}}, \tag{D.74}$$

where $s_t^{IS,*}$ is the RoW share of commodities use.[5] The specification in eq. (D.74) leads to optimality conditions for the demand for commodities as in the EA.

Intermediate goods. The intermediate good producers use labour and capital to manufacture domestic goods (non-commodity output) according to a Cobb-Douglas production function and are subject to a standard CES demand function of RoW output packers (analogously to the EA block):

$$Y_{i,t}^* = A_t^{Y,*} \left(cu_{i,t}^* K_{i,t-1}^*\right)^{\alpha_K} \left(N_t^*\right)^{\alpha}, \tag{D.75}$$

where $A_t^{Y,*}$ captures a trend in productivity, and $N_t^* = Actr_t^* POP_t^*$ is the active population in the economy. $K_{i,t-1}^*$ denotes capital. It is utilised at rate $cu_{i,t}^*$ and follows a law of motion analogous to that in the EA.

RoW adjustment costs are given by:

$$\Gamma_{i,t}^{P,*} = \frac{\sigma^{Y,*}\gamma^{P,*}}{2} Y_t^* \left(\frac{P_{i,t}^*}{P_{i,t-1}^*} - 1 - \pi^*\right)^2 \tag{D.76}$$

$$\Gamma_{i,t}^{cu,*} = \frac{P_t^{I,*}}{P_t^*} K_{i,t-1}^* \left(\gamma_0^{u,*}\left(cu_{i,t}^* - 1\right) + \frac{\gamma_1^{u,*}}{2}\left(cu_{i,t}^* - 1\right)^2\right) \tag{D.77}$$

$$\Gamma_{i,t}^{I,*} = \frac{\gamma_1^{I,*}}{2} \frac{P_t^{I,*}}{P_t^*} \frac{\left(I_{i,t}^* - I_{i,t-1}^* \exp\left(g^Y\right)\right)^2}{K_{t-1}^*} \tag{D.78}$$

The first-order condition with respect to $I_{i,t}^*$ reads:

$$\begin{aligned} Q_t^* = &\left(1 + \gamma_1^{I,*} \frac{\left(I_t^* - I_{t-1}^* \exp\left(g^Y\right)\right)}{K_{t-1}^*}\right) \\ &- E_t\left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{P_{t+1}^{I,*}}{P_t^{I,*}} \frac{P_t^*}{P_{t+1}^*} \gamma_1^{I,*} \frac{\left(I_{t+1}^* - I_t^* \exp\left(g^Y\right)\right)}{K_t^*} \exp\left(g^Y\right)\right], \end{aligned} \tag{D.79}$$

where $Q_t^* \equiv \frac{\mu_t^*}{\frac{P_t^{I,*}}{P_t^*}}$ is Tobin's marginal Q.

The first-order condition with respect to $K_{i,t}^*$ solves:

$$\begin{aligned} Q_t^* = &E_t\left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \frac{P_{t+1}^{I,*}}{P_{t+1}^*} \frac{P_t^*}{P_t^{I,*}} \left(\left(-\gamma_0^{u,*}\left(cu_{i,t+1}^* - 1\right) - \frac{\gamma_1^{u,*}}{2}\left(cu_{i,t+1}^* - 1\right)^2\right)\right.\right. \\ &\left.\left. + \left(1 - \delta^*\right) Q_{t+1}^* + \left(1 - \alpha^*\right) \mu_{t+1}^{Y,*} \frac{P_{t+1}^*}{P_{t+1}^{I,*}} \frac{Y_{i,t+1}^*}{K_{i,t}^*}\right)\right]. \end{aligned} \tag{D.80}$$

The first-order condition with respect to $cu_{i,t}^*$ yields:

$$\frac{P_t^{I,*}}{P_t^*} K_{i,t-1}^* \left(\gamma_0^{u,*} + \gamma_1^{u,*}\left(cu_{i,t}^* - 1\right)\right) = \mu_t^{Y,*}\left(1 - \alpha^*\right) \frac{Y_{i,t}^*}{cu_{i,t}^*}. \tag{D.81}$$

[5] Unlike the EA, the RoW region does not feature energy commodities as an additional (direct) element in households' consumption goods.

Price setting for non-oil output follows a New Keynesian Phillips curve (FOC for $P^*_{i,t}$)

$$\mu^{y,*}_t \sigma^{y,*} = (\sigma^{y,*} - 1) + \sigma^{y,*}\gamma^{P,*}\frac{P^*_t}{P^*_{t-1}}(\pi^*_t - \bar{\pi}^*)$$
$$- \sigma^{y,*}\gamma^{P,*}\left[\frac{1+\pi^*_{t+1}}{1+i^{s,*}_{t+1}}\frac{P^*_{t+1}}{P^*_t}\frac{Y^*_{t+1}}{Y^*_t}(\pi^*_{t+1} - \bar{\pi}^*)\right] + \sigma^{y,*}\varepsilon^{Y,*}_t, \tag{D.82}$$

where $\lambda^*_t = (C^*_t - h^*C^*_{t-1})^{-\theta^*}$ is the marginal utility of consumption, and $\varepsilon^{Y,*}_t$ is a cost push shock.

RoW commodity supply. A competitive sector supplies two distinct commodities, namely oil (o^*, Brent) and non-oil commodities (no^*, e.g. natural gas and materials) to domestic and foreign firms. There is a supply disturbance $\varepsilon^{IS,*}$ that captures exogenous commodity supply shocks, such as the discovery of new raw material deposits. Demand for commodities is determined by final good producers from the two regions (see above). A producer combines oil (o^*) and non-oil (no^*) commodities into the CES bundle IS^* that is exported to the EA or used locally with price $P^{IS,*}_t$:

$$P^{IS,*}_t = \varepsilon^{IS,*}_t \left[s^*(P^{o,*}_t)^{1-\sigma^{COMM,*}} + (1-s^*)(P^{no,*}_t)^{1-\sigma^{COMM,*}}\right]^{1/(1-\sigma^{COMM,*})}. \tag{D.83}$$

Commodity prices are exogenous in this model specification, i.e.:

$$P^{o,*}_t = \frac{P^*_t}{A^{o,*}_t}, \tag{D.84}$$

where $A^{o,*}_t$ is the exogenous oil-specific productivity technology (analogously for prices of non-oil commodities).

The total supply of commodities by the RoW is determined residually to satisfy global demand.

Consumption-savings choices. RoW households maximise utility subject to the aggregate budget constraint:

$$P^*_t Y^*_t + div^*_t = P^{C,*}_t C^*_t + P^{C,*}_t I^*_t + TB^*_t, \tag{D.85}$$

where div^*_t are dividends from intermediate good producers, and TB^*_t are net exports. I^*_t denotes investment. The consumption Euler equation is:

$$1 = E_t\left[\Lambda^*_{t,t+1}\frac{R^*_t}{1+\pi^{C,*}_{t+1}}\right], \tag{D.86}$$

where $\Lambda^*_{t,t+1} = \beta^* \exp(\varepsilon^{C,*}_t + \varepsilon^{tC,*}_t)\frac{\left(C^*_{t+1} - h^*C^*_t\right)^{-\theta^*}}{\left(C^*_t - h^*C^*_{t-1}\right)^{-\theta^*}}$.

D.8 Exogenous shocks

The stochastic processes follow the same decomposition as in the main text. Since the model is estimated on non-stationary data, it allows for both low-frequency and business-cycle movements in the exogenous driving forces.

For the main non-stationary variables—namely, productivity and home-bias terms—the logarithm of the process is written as the sum of two components:

$$\ln A_t^q = T_t^q + S_t^q, \tag{D.87}$$

where T_t^q denotes the low-frequency component and S_t^q a second component that is stationary in the generic case.

The low-frequency component evolves through its first difference,

$$g_t^q \equiv T_t^q - T_{t-1}^q, \tag{D.88}$$

which follows an autoregressive process:

$$g_t^q = \rho_g^q g_{t-1}^q + (1 - \rho_g^q)\bar{g}^q + u_t^{g,q}, \qquad 0 < \rho_g^q < 1, \tag{D.89}$$

where $\bar{g}^q$ is the long-run growth rate and $u_t^{g,q}$ is referred to as a *trend-growth-rate shock*.

In the generic case, the second component follows an AR(1) process in levels:

$$S_t^q = \rho^q S_{t-1}^q + u_t^q, \qquad 0 \leq \rho^q < 1, \tag{D.90}$$

where u_t^q is a *transitory shock*. The innovations $u_t^{g,q}$ and u_t^q are assumed to be orthogonal i.i.d. white-noise processes.

This specification implies that A_t^q follows a unit-root process with stochastic drift. In the generic case, the process combines persistent movements in trend growth with transitory level deviations around that drifting trend.

For intermediate-goods productivity, however, we depart from the generic case and set the persistence of the second component to unity. As in the main text, this implies that intermediate-goods productivity growth is the sum of a trend-growth component and a white-noise disturbance:

$$\Delta \ln A_t^Y = g_t^Y + u_t^Y. \tag{D.91}$$

Thus, for intermediate-goods productivity, the second component is no longer stationary, consistent with the near-unit-root behavior emphasized in the paper.

Stationary exogenous processes All remaining exogenous variables that do not feature a stochastic trend are modeled as stationary AR(1) processes in levels. A generic stationary forcing variable ε_t^x evolves according to

$$\varepsilon_t^x = \rho^x \varepsilon_{t-1}^x + u_t^x, \qquad 0 \leq \rho^x < 1, \tag{D.92}$$

where u_t^x is an i.i.d. white-noise innovation.

E Data Sources

E.1 Overview: Countries and variables

The analysis uses quarterly and annual data for the period 1998q4 to 2024q2 based on the dataset of the European Commission's Global Multi-country Model (Albonico et al., 2019). Data for the EA aggregate (EA20) are taken from Eurostat, in particular, from the European System of National Accounts. The Rest of the World (RoW) data are annual data and are constructed using IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

Series for GDP and prices in RoW start in 1999 and are constructed on the basis of data for the following 57 countries: Albania, Algeria, Argentina, Armenia, Australia, Azerbaijan, Belarus, Brazil, Bulgaria, Canada, Chile, China, Colombia, Czech Republic, Denmark, Egypt, Georgia, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Israel, Japan, Jordan, Korea, Lebanon, Libya, FYR Macedonia, Malaysia, Mexico, Moldova, Montenegro, Morocco, New Zealand, Nigeria, Norway, Philippines, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Syria, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, United States and Venezuela.

When quarterly-frequency data are not available, mixed-frequency data estimation is used. In this case, we assign annual RoW data to the last quarters of each year (as quarterly annualized variables) and allow the Kalman smoother to interpolate the annual data in the other three quarters in a model-consistent manner.

Table E.1 lists the observed time series. GDP deflators and relative prices of aggregates are computed as the ratios of current price value to chained indexed volume.

E.2 Latent trend productivity

Estimating the latent productivity trend requires care, as the signal-to-noise ratio between permanent and transitory components must be specified so that the trend does not absorb cyclical fluctuations. If the variance or persistence of the trend shock is too large, the Kalman filter may attribute short-run movements in output to permanent productivity, generating spurious "cyclical TFP" instead of allowing business-cycle dynamics to be captured by transitory shocks and endogenous adjustment mechanisms (e.g., variable capacity utilization) in the model.

We address this identification issue in two complementary ways. First, we impose informative priors on the persistence and standard deviation of the productivity trend growth process (reported in the main text and Appendix F), ensuring that the trend captures low-frequency movements rather than medium-run fluctuations.

Second, we anchor the EA productivity trend using external proxy information, namely the annual EA TFP trend series published by the European Commission (Havik et al., 2014).[6]

We have also estimated alternative specifications that exclude the proxy series and rely solely on tighter prior restrictions on the signal-to-noise ratio which yield very similar productivity trends. Nevertheless, we retain the specification with the external proxy, as it provides a transparent way

[6] Available at https://circabc.europa.eu/ui/group/671d465b-0752-4a2e-906c-a3effd2340ba.

Euro Area
Initial conditions
Log of total capital stock (initial level)
Supply side
Log of population
Log of active population rate
Log of TFP trend
Macro aggregates
Log of real GDP
Log of nominal GDP
Log of real consumption
Log of nominal consumption
Log of real total investment
Log of nominal total investment
Net foreign assets
Labour market
Log of hours worked
Log of wages
Government
Log of government debt
Log of government interest payments
Log of real government consumption
Log of nominal government consumption
Log of nominal government investment
Log of transfers
Nominal policy interest rate
External sector
Log of real exports
Log of nominal exports
Log of real import volumes
Log of nominal import volumes
Log of nominal effective exchange rate
Commodities
Log of nominal commodity import volumes
Oil price (Brent)
Raw material prices
Rest of the World
Demographics
Log of population
Macro aggregates and prices
Log of nominal GDP (quarterly annualised)
Log of real GDP (quarterly annualised)
Log of nominal investment share (quarterly annualised)
Oil price (Brent)
Nominal policy interest rate (quarterly annualised)

Table E.1: Data used for estimation (list of observables).

to discipline the low-frequency component.

Figure E.1 reports the smoothed EA trend productivity growth together with the proxy used as an observable in the estimation, constructed by interpolating annual data to quarterly frequency.

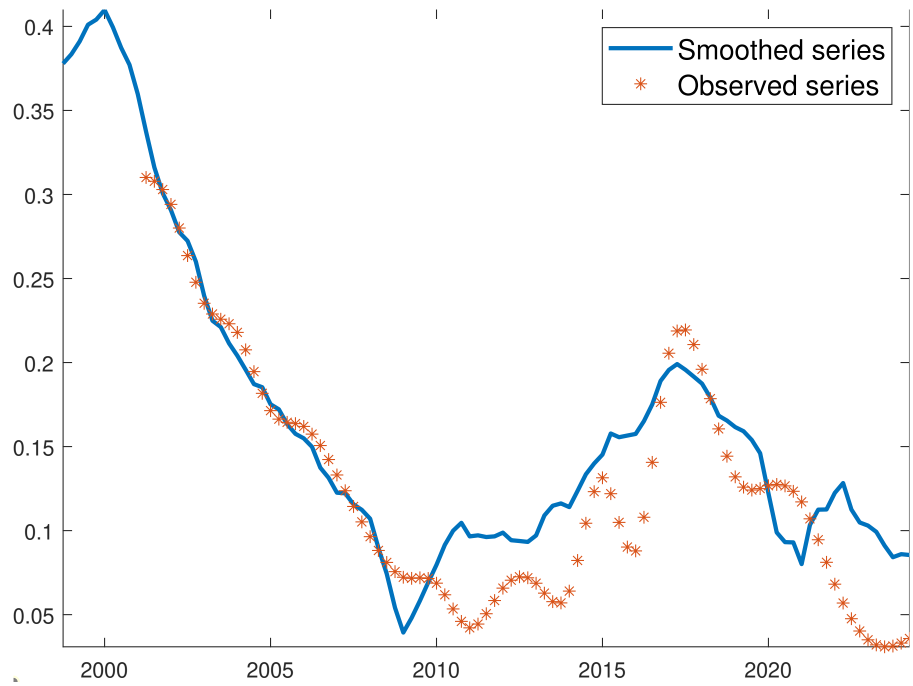


Figure E.1: Smoothed versus observed series of trend productivity growth

E.3 Construction of extra-EA trade series

EA national accounts report aggregate imports and exports that include intra-EA trade. However, the model requires trade flows vis-à-vis the rest of the world only. Since quarterly real extra-EA trade series are not directly available, we reconstruct nominal and real extra-EA trade data as follows.

Let total trade including intra-EA flows be:

$$X^{N,x+intra} = X^{x+intra} P^{X,x+intra}, \tag{E.1}$$

$$M^{N,x+intra} = M^{x+intra} P^{M,x+intra}. \tag{E.2}$$

The nominal trade balance is

$$TB = X^{N,x+intra} - M^{N,x+intra}. \tag{E.3}$$

By construction, intra-EA trade cancels in the trade balance.

Because intra-EA trade is only available in nominal terms, real extra-EA trade series must be constructed indirectly.

Nominal exports (extra-EA). We compute the intra-EA share of exports:

$$S_X^{intra} = \frac{X^{N,intra}}{X^{N,x+intra}}. \tag{E.4}$$

Nominal exports to the rest of the world are then:

$$X^N = (1 - S_X^{intra}) X^{N,x+intra}. \tag{E.5}$$

These data include goods and services. In the model, the EA does not export industrial supplies (energy commodities and raw materials), so exported goods are interpreted as non-commodity goods.

Nominal imports (extra-EA). Nominal imports excluding intra-EA trade are obtained residually:

$$M^N = X^N - TB. \tag{E.6}$$

In the model, total imports are decomposed into non-commodity imports and industrial supplies:

$$M^N = M_G^N + IS^N, \tag{E.7}$$

where IS^N denotes industrial supplies (energy commodities and raw materials). Consistent with the model structure, industrial supplies are assumed to be sourced from outside the EA; intra-EA trade therefore concerns non-commodity goods and services only.

Export prices. National accounts do not provide separate quarterly deflators for intra- and extra-EA exports. We therefore assume

$$P^X = P^{X,x+intra}, \tag{E.8}$$

which ensures consistency between nominal and real aggregates.

Import prices. We assume that the (unobserved) intra-EA import deflator equals the EA export deflator:

$$P^{M,intra} = P^X. \tag{E.9}$$

The total import deflator including intra-EA trade satisfies the nominal-share-weighted aggregation:

$$P^{M,x+intra} = P^X S_M^{intra} + P^M(1 - S_M^{intra}), \tag{E.10}$$

where

$$S_M^{intra} = \frac{M^{N,intra}}{M^{N,x+intra}}, \qquad M^{N,intra} = M^{N,x+intra} - M^N. \tag{E.11}$$

Solving for the extra-EA import deflator gives

$$P^M = \frac{P^{M,x+intra} - P^X S_M^{intra}}{1 - S_M^{intra}}. \tag{E.12}$$

Real trade flows. Real extra-EA trade series are constructed as

$$X = \frac{X^N}{P^X}, \qquad M = \frac{M^N}{P^M}. \tag{E.13}$$

In sum, because quarterly real extra-EA trade data are unavailable, we reconstruct them using nominal intra-EA shares and internally consistent price aggregation. The procedure relies on identical deflators for intra- and extra-EA exports, a nominal-share-weighted aggregation of import prices, and the assumption that industrial supplies are sourced from outside the EA.

F Calibration and estimation results

F.1 Calibration

We calibrate a subset of structural parameters to match long-run averages, institutional features, and steady-state ratios of the EA economy. The calibrated values are reported in Table F.1.

		EA
Households		
Preference for government bonds	α^B	-0.0015
Preference for stocks	α^S	0.0066
Preference for foreign bonds	α^{BW}	0.0073
Intertemporal discount factor	β	0.9996
Savers share	ω^S	0.6700
Import share in consumption	$s^{M,C}$	0.1012
Import share in investment (private and gov)	$s^{M,I}$	0.1539
Import share in exports	$s^{M,X}$	0.1432
Weight of disutility of labor	s^N	2.2497
Production		
Cobb-Douglas labor share	α	0.6500
Depreciation of private capital stock (%, annual)	δ	5.7162
Share of oil in total output	s^{Oil}	0.0375
Linear capacity utilisation adj. costs	$\gamma^{u,1}$	0.0307
Value-added demand elasticity	σ^y	9.7189
Monetary and fiscal policy		
CPI inflation target (%, annual)	$\bar{\phi}^{c,vat}$	2.0000
Interest rate persistence	ρ^i	0.9009
Consumption tax	τ^C	0.2000
Corporate profit tax	τ^k	0.3000
Labour tax	τ^N	0.4269
Deficit target	def^T	0.0295
Debt target	$\bar{BG}$	3.1087
Long-run growth and ratios		
Population trend growth (% annual)	g_{pop}	0.3368
Size of EA in world GDP	$size_{EA}$	18.1598
Private consumption share in SS	C/Y	0.5751
Private investment share in SS	I/Y	0.1798
Govt consumption share in SS	C^G/Y	0.2076
Govt investment share in SS	I^G/Y	0.0326
Transfers share in SS	T/Y	0.1641

Table F.1: Calibration of key model parameters

Trend inflation is set to 2 percent annually, consistent with the ECB's price stability objective. Population growth is set to match the average annual growth rate observed in the data. The steady-state nominal interest rate is implied by the intertemporal discount factor, together with trend inflation and output growth.

On the household side, the share of saver households is calibrated in line with micro evidence reported in Dolls et al. (2012). Portfolio preference parameters are chosen to reproduce steady-state asset positions, while the weight of labor disutility ensures plausible steady-state hours worked. The import content of consumption, investment, and exports is calibrated using input–output evidence following Bussiere et al. (2013).

On the production side, the Cobb–Douglas labor share, α, is set to 0.65, consistent with national accounts evidence. The depreciation rate matches average annual capital depreciation. The steady-state share of industrial supplies (commodities) in output corresponds to the average ratio of imported commodities to GDP for the EA (around 4 percent). For RoW region, this measure includes domestically produced commodities (around 5 percent) that are domestically consumed and exported to euro area. Since the RoW block does not feature fiscal policy, the weight of investment in the production function is calibrated to deliver the same increasing return to scale as for the EA block ($\alpha_K^{RoW} = 0.45$). Fiscal policy parameters are calibrated to match average effective tax rates and steady-state fiscal ratios. The deficit and debt targets reflect observed fiscal positions over the sample period. Finally, steady-state expenditure shares of private consumption, investment, government demand, and transfers replicate their historical averages relative to GDP.

F.2 Posterior estimates

The remaining structural parameters and shock processes are estimated using Bayesian full-information methods applied to the linearized model. Prior assumptions and posterior distributions are reported in Tables F.2 and F.3.

		Prior			Posterior		
		Distr.	Mode	Std.	Mode	HPD10	HPD90
Preferences and technology							
Consumption habit persistence	h	Beta	0.50	0.10	0.85	0.82	0.91
Risk aversion	θ	Gamma	1.50	0.20	1.54	1.35	1.88
Inverse Frisch elasticity of labour supply	θ^N	Gamma	2.50	0.50	2.36	1.77	2.89
Import price elasticity	σ^Z	Gamma	2.00	0.40	2.01	1.66	2.98
Commodity elasticity of substitution in production	σ^O	Beta	0.10	0.04	0.08	0.04	0.14
Nominal and real frictions							
Price adjustment cost	γ^P	Gamma	20.00	12.00	30.11	18.17	36.69
Nominal wage adjustment cost	γ^W	Gamma	40.00	24.00	29.48	8.75	39.07
Real wage rigidity	γ^{WR}	Beta	0.85	0.06	0.91	0.82	0.92
Hours adjustment cost	γ^H	Gamma	20.00	12.00	4.78	3.92	7.85
Employment adjustment cost	γ^E	Gamma	20.00	12.00	15.89	12.65	19.33
Capacity utilisation adjustment cost	$\gamma^{U,2}$	Gamma	0.03	0.01	0.00	0.00	0.01
Investment adjustment cost	$\gamma^{I,2}$	Gamma	200.00	150.00	1088.30	649.75	1669.17
Import demand inertia	ρ^Z	Beta	0.50	0.20	0.93	0.80	0.97
Commodity demand inertia	ρ^O	Beta	0.50	0.20	0.06	0.03	0.16
Policy							
Interest rate persistence	ρ^i	Beta	0.85	0.08	0.90	0.88	0.91
Taylor rule response to inflation	$\eta^{i,\pi}$	Inv.Gamma	1.70	0.15	1.54	1.45	1.74
Taylor rule response to output gap	$\eta^{i,y}$	Beta	0.05	0.02	0.09	0.05	0.10
Deficit coefficient in transfer rule	$\eta^{i,d}$	Beta	0.03	0.01	0.03	0.02	0.04
RoW region							
Consumption habit persistence	h^*	Beta	0.70	0.10	0.74	0.61	0.80
Risk aversion	θ^*	Gamma	1.50	0.20	1.35	1.15	1.67
Import price elasticity	$\sigma^{C,*}$	Gamma	2.00	0.40	1.76	1.32	1.98
Price adjustment cost	$\gamma^{P,*}$	Gamma	20.00	12.00	27.09	21.09	51.19
Capacity utilisation adjustment cost	$\gamma^{U,2,*}$	Gamma	0.03	0.01	0.03	0.01	0.04
Investment adjustment cost	$\gamma^{I,2,*}$	Gamma	200.00	150.00	390.08	319.08	860.64
Import demand inertia	$\rho^{Z,*}$	Beta	0.50	0.20	0.95	0.81	0.97
Interest rate persistence	$\rho^{i,*}$	Beta	0.85	0.08	0.90	0.86	0.94
Response to inflation	$\eta^{i,\pi,*}$	Inv.Gamma	1.70	0.15	1.67	1.47	1.81
Response to output gap	$\eta^{i,y,*}$	Beta	0.20	0.08	0.19	0.09	0.33
Persistence of population growth trend	$g^{pop,*}$	Beta	0.85	0.07	0.99	0.98	0.99

Table F.2: Prior and posterior distributions of key estimated model parameters

		Prior			Posterior		
		Distr.	Mode	Std.	Mode	HPD10	HPD90
Autocorrelations of forcing variables							
UIP shock EA	ρ^{bw}	Beta	0.50	0.20	0.76	0.65	0.87
Net foreign asset position EA	ρ^{nfa}	Beta	0.50	0.20	0.87	0.81	0.93
Government consumption EA	ρ^{G}	Beta	0.70	0.10	0.95	0.91	0.97
Government investment EA	ρ^{IG}	Beta	0.70	0.10	0.93	0.90	0.96
Transfers EA	ρ^{T}	Beta	0.70	0.10	0.88	0.83	0.94
Lump-sum tax EA	ρ^{TAX}	Beta	0.85	0.06	0.89	0.84	0.95
Consumption-specific productivity EA	ρ^{C}	Beta	0.50	0.20	0.92	0.87	0.98
Government-consumption specific productivity EA	ρ^{G}	Beta	0.50	0.20	0.93	0.89	0.97
Investment-specific productivity EA	ρ^{I}	Beta	0.50	0.20	0.96	0.93	0.98
Export productivity EA	$\rho^{g,X}$	Beta	0.85	0.10	0.70	0.58	0.80
Labor demand EA	ρ^{ND}	Beta	0.50	0.20	0.74	0.60	0.83
Discount factor EA	ρ^{C}	Beta	0.50	0.20	0.86	0.73	0.89
Preference for government bonds EA	ρ^{g}	Beta	0.50	0.20	0.95	0.91	0.97
Preference for stocks (risk premia) EA	ρ^{S}	Beta	0.85	0.07	0.91	0.86	0.93
Stationary home bias shock EA	ρ^{M}	Beta	0.50	0.10	0.61	0.50	0.73
Risk shock RoW	$\rho^{C,*}$	Beta	0.50	0.20	0.69	0.58	0.80
Stationary home bias shock RoW	$\rho^{M,*}$	Beta	0.50	0.10	0.64	0.53	0.72
Stationary export productivity RoW	$\rho^{g,X,*}$	Beta	0.50	0.10	0.62	0.50	0.75
Standard deviations (%) of innovations to forcing variables							
UIP shock EA	σ^{bw}	Gamma	1.00	0.40	0.49	0.19	0.61
Net foreign asset position EA	σ^{nfa}	Gamma	1.00	0.40	1.77	1.60	2.00
Government consumption EA	σ^{G}	Gamma	1.00	0.40	0.13	0.12	0.15
Government investment EA	σ^{IG}	Gamma	1.00	0.40	0.08	0.07	0.09
Transfers EA	σ^{T}	Gamma	1.00	0.40	0.17	0.15	0.19
Lump-sum tax EA	σ^{TAX}	Gamma	1.00	0.40	0.57	0.52	0.66
Consumption-specific productivity EA	σ^{C}	Gamma	1.00	0.40	0.26	0.24	0.31
Government-consumption specific productivity EA	σ^{G}	Gamma	1.00	0.40	0.65	0.59	0.73
Investment-specific productivity EA	σ^{I}	Gamma	1.00	0.40	0.32	0.28	0.38
Stationary export productivity EA	σ^{X}	Gamma	0.10	0.04	0.42	0.25	0.50
Labor demand EA	σ^{ND}	Gamma	0.50	0.20	1.27	1.03	1.72
Discount factor EA	σ^{β}	Uniform	2.00	1.15	0.55	0.31	1.50
Preference for government bonds EA	σ^{g}	Gamma	1.00	0.40	0.10	0.09	0.11
Preference for stocks (risk premia) EA	σ^{S}	Uniform	0.75	0.43	0.48	0.30	0.98
Stationary home bias shock EA	σ^{M}	Gamma	1.00	0.40	2.42	2.11	2.68
Risk shock RoW	$\sigma^{risk,*}$	Gamma	1.00	0.40	0.67	0.37	1.02
Stationary home bias shock RoW	$\sigma^{M,*}$	Gamma	1.00	0.40	2.45	2.19	2.81
Stationary export productivity RoW	$\sigma^{X,*}$	Gamma	1.00	0.40	1.65	1.42	1.84

Table F.3: Prior and posterior distributions of key estimated shock processes (in %)

On the preference side, the posterior indicates a high degree of external habit formation, implying substantial consumption smoothing. Risk aversion is estimated within conventional ranges in the literature and remains close to its prior mean. The inverse Frisch elasticity implies moderate labor supply elasticity. The import price elasticity exceeds two, suggesting a relatively elastic response of trade volumes to relative price movements, while the elasticity of substitution in commodity use remains low.

Regarding nominal and real rigidities, the data support substantial price and wage adjustment costs. Real wage persistence is estimated to be high, contributing to inertia in real marginal costs. Employment adjustment costs are economically meaningful, while hours adjustment costs are more moderate, implying that firms adjust along both intensive and extensive margins. Investment adjustment costs are large, reinforcing gradual capital accumulation dynamics. Demand for imports exhibits strong persistence, consistent with the sluggish adjustment of trade flows observed in the data, while for commodities persistence is small.

Monetary policy is characterized by substantial interest rate smoothing. The response to inflation is consistent with the Taylor principle, while the response to the output gap is positive but modest. The fiscal transfer rule reacts moderately to deviations of the deficit from its target.

G Historical decompositions using real-time filtered shocks

The historical decompositions reported in the main body of the paper, such as those shown in Figure 8, are computed using smoothed shocks. In the presence of permanent shocks, however, filtered shocks may differ from smoothed shocks, potentially affecting inference on the role of permanent growth shocks for the real exchange rate and the trade-balance-to-GDP ratio.

To assess the quantitative relevance of this issue, we replicate the analysis in Figure 8 using *real-time* (one-sided) shock decompositions. These decompositions are based on filtered shocks that use only information available up to time t, rather than on the full sample. Figures G.1 and G.2 show that the main results are robust when filtered shocks are used instead of smoothed shocks.

Operationally, the one-sided shock decompositions are obtained by computing rolling-window smoothers and associated decompositions over expanding samples spanning the interval $t = 1, \ldots, T$. For each period t, the smoother and corresponding decomposition are computed using data from periods 1 through t, and the resulting shock contributions for period t are stored. Plotting these last-period contributions for $t = 1, \ldots, T$ yields the time series reported in Figures G.1 and G.2. By construction, these decompositions rely exclusively on past information and therefore constitute proper real-time (one-sided) decompositions.

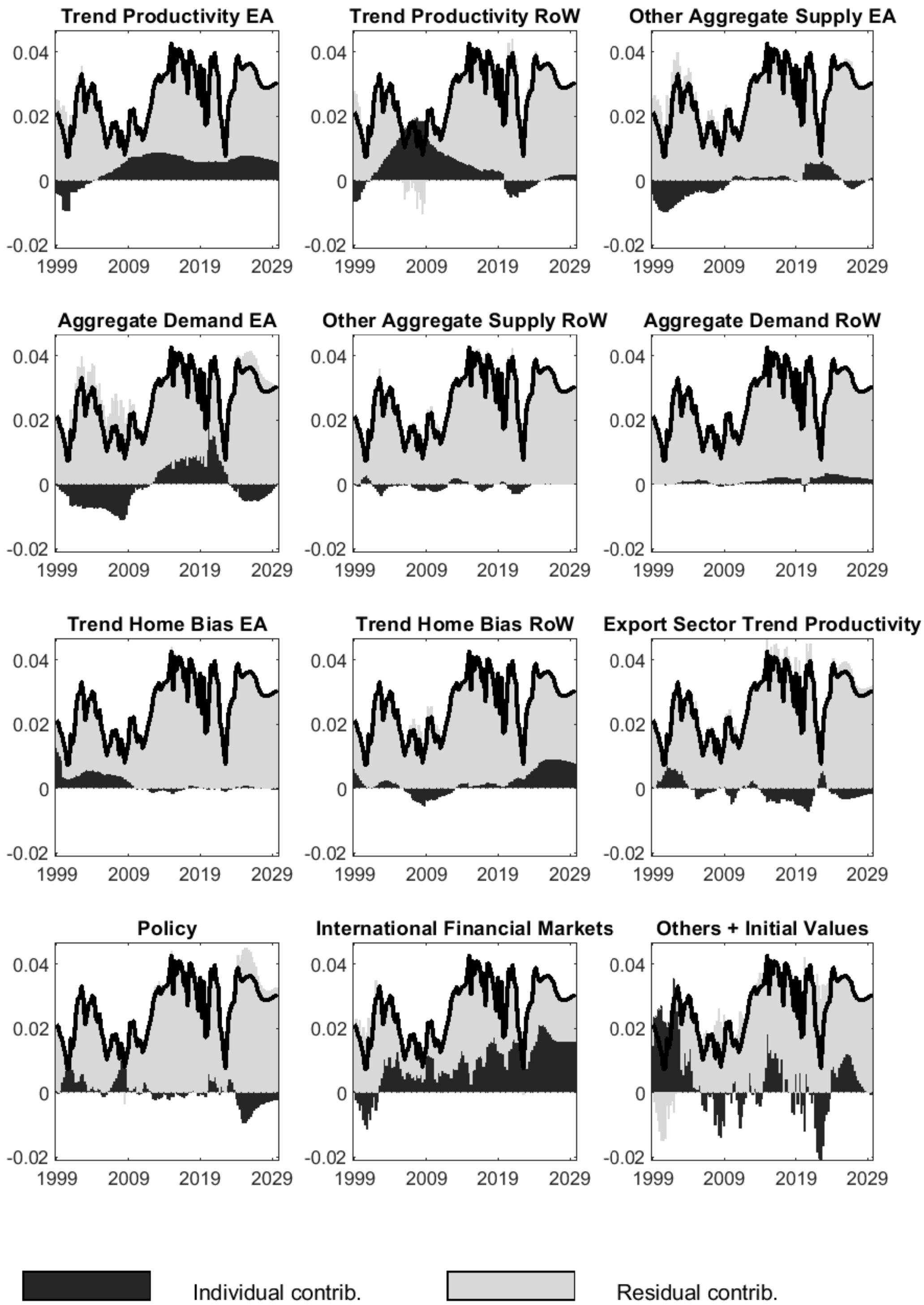


Figure G.1: Real-time shock decomposition: trade-balance-to-GDP ratio (EA)

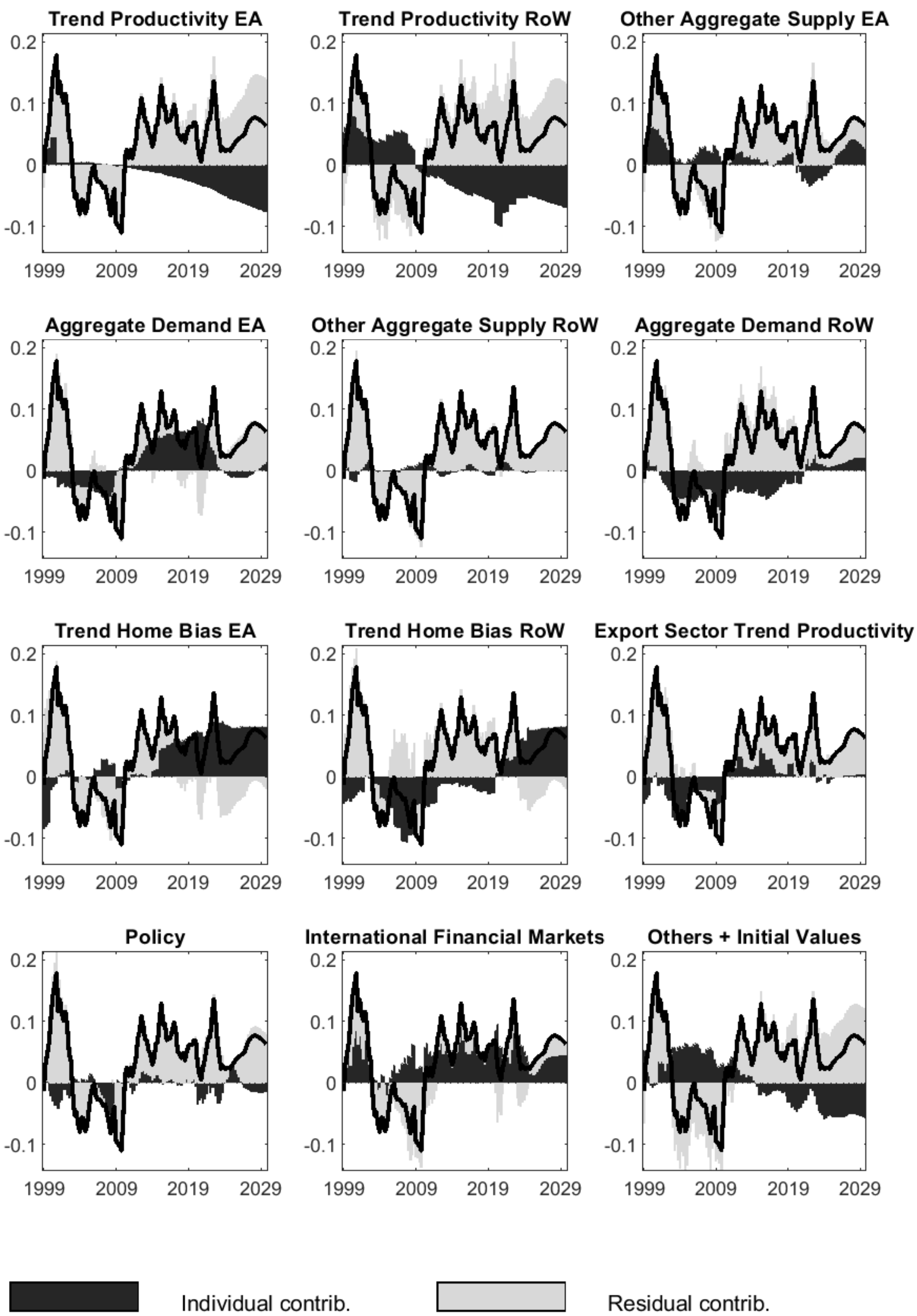


Figure G.2: Real-time shock decomposition: real exchange rate (EA)

H Model solution and approximation

H.1 Balanced growth

We assume the existence of global deterministic trends in population, labour productivity, and prices. In particular, global inflation is set to 2 percent per year, and long-run growth is driven by exogenous trends in productivity and population. Population growth g_{pop} is set to euro area average (0.34% annual), while labour productivity growth g_a is estimated.

For each country j, the intermediate sector productivity parameter evolves as:

$$A^y_{j,t} = (A^y)^t \exp\big(T^y_{j,t} + S^y_{j,t}\big), \tag{H.1}$$

where $A^y = \exp(g_a)$ is the global deterministic (log-linear) growth of productivity, and $T^y_{j,t}$ and $S^y_{j,t}$ denote the non-stationary and stationary country-specific productivity shocks.

Population in each country j evolves as:

$$\text{Pop}_{j,t} = \text{Pop}^t \exp\big(T^{\text{pop}}_{j,t}\big), \tag{H.2}$$

where $\text{Pop} = \exp(g_{\text{pop}})$ is the global population log-linear growth trend.

Country-specific stochastic trends of productivity $T^y_{j,t}$ and population $T^{\text{pop}}_{j,t}$ do not have any additional drift term, hence only the global productivity and population trends matter for determining balanced growth path of the model, which we can derive starting from the production function (that also includes public capital):

$$Y_{j,t} = (A^y_{j,t} \cdot N_{j,t})^{\alpha} K^{1-\alpha}_{j,t} KG^{1-\alpha_G}_{j,t}. \tag{H.3}$$

Imposing common growth for GDP, private and public capital and remembering that hours grow with population, we get

$$Y = (A^y \cdot \text{Pop})^{\alpha} Y^{1-\alpha} Y^{1-\alpha_G}, \tag{H.4}$$

from which the balanced real GDP growth rate is:

$$g_y = \frac{\alpha}{\alpha + \alpha_G - 1}\,(g_{\text{pop}} + g_a). \tag{H.5}$$

Global GDP annual growth g_y is $\approx 1.8\%$ at posterior mode, given calibrated labor share $\alpha = 0.65$ and public capital share $1 - \alpha_G = 0.1$.

The de-trended model around this balanced growth path provides the baseline stationary solution for all model variables (levels and growth rates). The model is linearized around this baseline stationary solution. For convenience and without loss of generality, the model also features some normalization assumptions:

- baseline GDP level is normalized to 1 for all regions,
- baseline price levels are normalized to 1 for all regions, and so are the nominal exchange rates,

- country size weights used in all cross-country market clearing equations are calibrated based on the average detrended real GDP data weighted by the respective USD/national currency (NC) exchange rate in base year (for proper comparability, all GDP sizes need to be converted into a common currency).

Consistently with model balanced growth and normalization assumptions, data series are detrended using deterministic balanced growth trends. GDP size for each country is set equal to average of detrended real GDP weighted by USD/NC exchange rate in base year. Then, all real variables are normalized by the GDP scale, so that data have the same normalization of the model.

Similarly, all price levels are first detrended by the 2% global inflation rate and then rescaled to be 1 in the base year. All exchange rates are also normalized to be 1 in the base year.

The model has 11 unit roots:

- 2 population trends and one active population shock for euro area,
- 2 productivity trend (both RW and AR(1) growth shocks),
- 2 export productivity shocks,
- 2 import share shocks,
- P level EA,
- P level RoW.

All growth rates, inflation rates and interest rates are stationary: this implies that, whatever non-stationary shock occurs, the model will ultimately converge to the assumed/estimated balanced growth path. At the same time, unit roots imply that all level variables (real/nominal) are non-stationary, i.e., such variables will converge to a new level relative to the balanced growth path, hence relative sizes of the different regions also permanently change. Price ratios, the RER, real/nominal ratios also permanently adjust after such unit root shocks. Co-integration relationship among nominal variables still occur via stationarity of TB and NFA to nominal GDP shares (so that the RER permanently adjusts to ensure this).

The new level reached by the linearized model is an approximation of the one that would be obtained via a perfect foresight simulation using the original non-linear model. Such an approximation is measurable and testable, comparing impulse-response functions of the linear and non-linear models.

H.2 Simulation approximation error

Using estimated standard deviations for the innovations of such non-stationary processes, linear and non-linear results are almost identical. It is interesting to understand key permanent adjustments associated to such non-stationary shocks, and whether the linearized model is able to capture the correct changes with respect to the nonlinear one. These are shown in Table H.1, where we simulate the model using innovations ten times larger than the estimated standard deviations, in order to increase the effect of the nonlinearity. Deterministic and linear simulations are still the same in the short run, while the terminal level may differ up to 10-15%, i.e. the linear model still approximates the nonlinear solution reasonably well even in the presence of sizable permanent level shifts.

Table H.1: Simulation approximation error: multipliers after 10 std permanent level shocks

	Linear				Nonlinear (positive (negative) shock)			
Shock	**GDP EA**	**GDP RoW**	**RER EA**	**TB EA**	**GDP EA**	**GDP RoW**	**RER EA**	**TB EA**
Persistent productivity (EA)	1.14	0.01	0.62	0.04	1.16 (1.13)	0.01	0.57 (0.69)	0.03 (0.05)
Level productivity (EA)	1.14	0.01	0.59	0.02	1.14	0.01	0.59 (0.59)	0.02
Persistent productivity (RoW)	0.08	1.17	-0.59	-0.03	0.08 (0.09)	1.19 (1.15)	-0.52 (-0.69)	-0.03 (-0.04)
Level productivity (RoW)	0.08	1.17	-0.56	-0.02	0.08	1.17	-0.56 (-0.57)	-0.02
Export productivity (EA)	-0.08	-0.02	0.26	-0.02	-0.08	-0.02 (-0.02)	0.26 (0.25)	-0.02
Export productivity (RoW)	-0.12	-0.05	-0.34	-0.00	-0.11 (-0.13)	-0.05 (-0.06)	-0.35 (-0.34)	-0.00
Import share (EA)	-0.07	0.01	0.39	-0.00	-0.07	0.01	0.41 (0.37)	-0.01 (-0.00)
Import share (RoW)	0.08	-0.01	-0.45	0.01	0.08	-0.02 (-0.00)	-0.45	0.01
Population (EA)	1.14	0.01	0.59	0.02	1.15	0.01	0.58 (0.60)	0.02
Population (RoW)	0.08	1.17	-0.50	0.00	0.08	1.18 (1.17)	-0.49 (-0.51)	0.00
Active population rate (EA)	1.14	0.01	0.57	0.02	1.15 (1.14)	0.01	0.57 (0.58)	0.02

Note: Multipliers are computed as the effect after 1000 periods.

H.3 Data filtering approximation error

One may also measure the approximation error of data filtering using a linearized model around balanced growth path. To do so, we perform counterfactual non-linear simulations using the linear smoother to set initial state variables and historical shocks, with a staggered type of algorithm:

Algorithm 1 Staggered algorithm for counterfactual non-linear simulations (data filtering approximation error)

```
1: for t = 1, ..., T do
2:     Given previous-period states s_{t-1} and current-period shocks ε_t,
       solve a deterministic perfect-foresight simulation for many periods ahead until convergence.
3:     From that simulation, record current-period simulated values y_t and states s_t.
4:     Use s_t to initialize period t + 1.
5: end for
```

We report results of this exercise in Figure H.1: the non-linear simulations broadly reproduce the correct historical patterns. One notable exception regards the Covid period during which the magnitude of the shocks hitting the economy was several times larger than usual business cycle shocks and hence pushed (temporarily) the model far from the baseline approximation region. Would these discrepancies be reduced by using stationary persistent shocks in place of the nonstationary processes? The answer is negative, since the approximation error depends on how much the data deviate from the balanced growth path, independently on the shocks that generate these deviations: indeed it is rather the pandemic shock (an i.i.d. shock) that triggers the major wedge between perfect foresight and first order simulations. The nature of the shocks only affect the forecast error and the long run implications of those shocks, but cannot change the deviation of the data with respect to the assumed balanced growth.

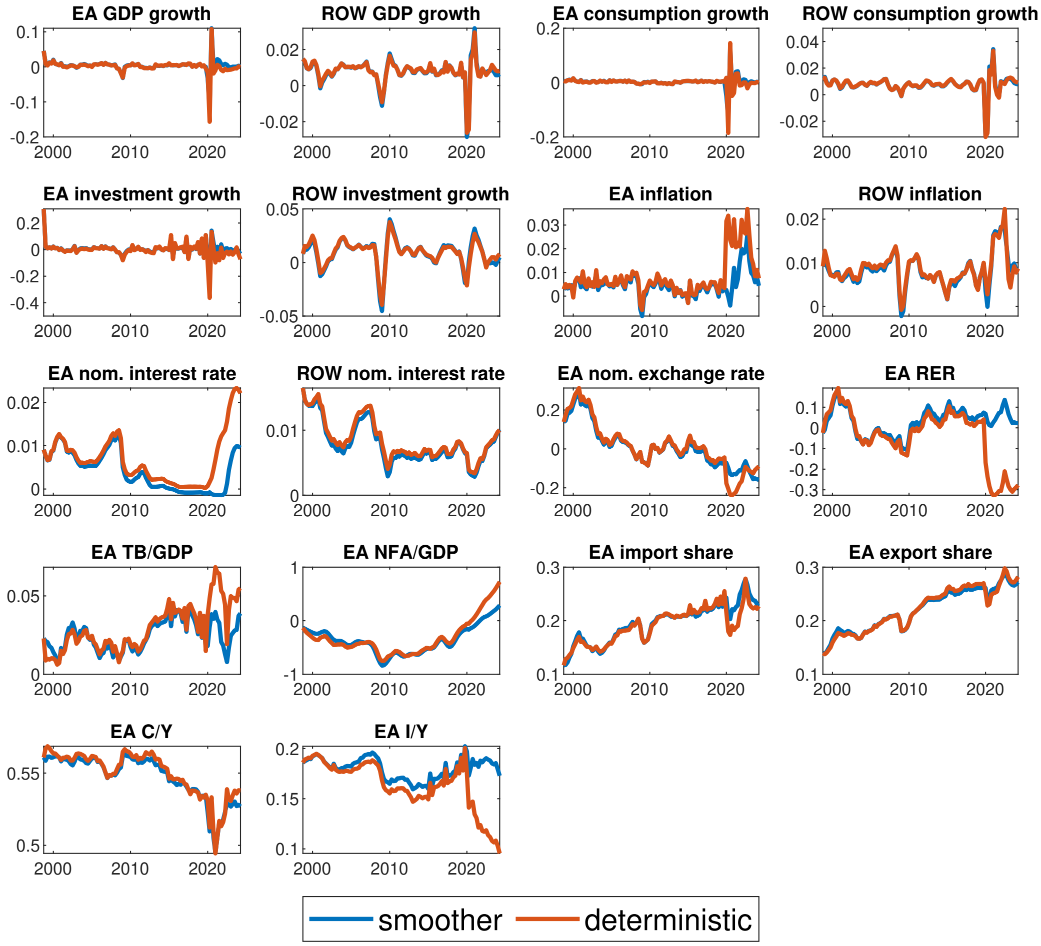


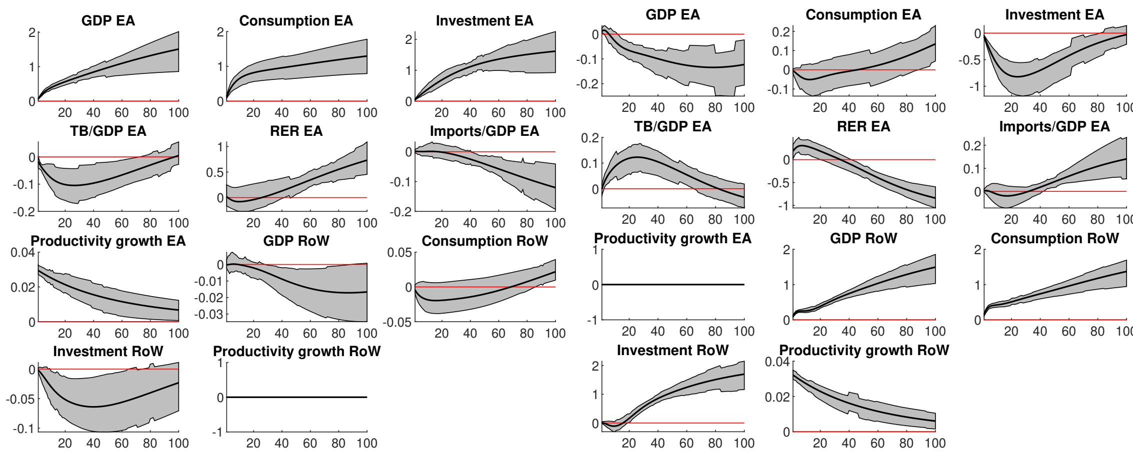
Figure H.1: Counterfactual non-linear simulations vs linear approximation

I Parameter uncertainty

This appendix reports impulse-response functions and historical shock decompositions, together with estimation uncertainty based on posterior distributions and parameter uncertainty.

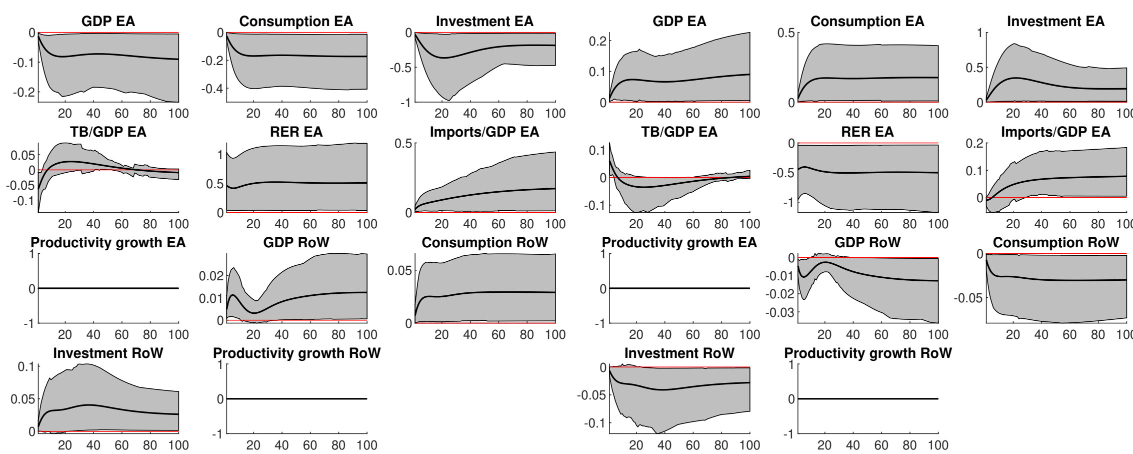
I.1 Uncertainty in the transmission of trend growth shocks

Figures I.1–I.3 present impulse-response functions to the main permanent shocks with 90% confidence intervals that account for posterior parameter uncertainty.



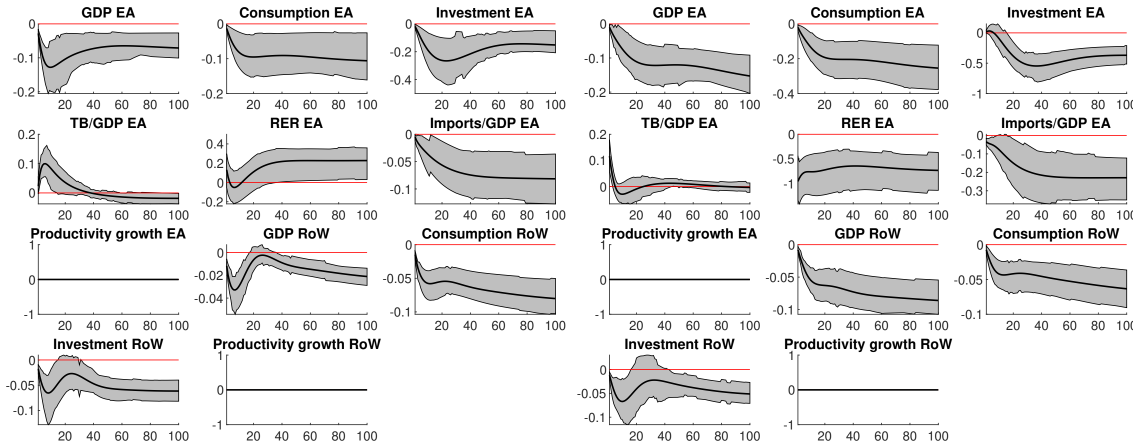
(a) IRF to EA aggregate productivity trend growth rate shock
(b) IRF to RoW aggregate productivity trend growth rate shock

Figure I.1: Uncertainty impulse-response functions (UIRFs): productivity growth shocks



(a) IRF to EA trend home bias shock
(b) IRF to RoW trend home bias shock

Figure I.2: Uncertainty impulse-response functions (UIRFs): home bias shocks (negative)



(a) IRF to EA export sector productivity trend growth rate shock

(b) IRF to RoW export sector productivity trend growth rate shock

Figure I.3: Uncertainty impulse-response functions (UIRFs): export productivity shocks

I.2 Uncertainty of shock contributions

To quantify estimation uncertainty in the historical shock decompositions, we draw repeatedly from the posterior distribution of model parameters and, for each draw, recover the corresponding sequence of structural shocks using the Kalman smoother. Conditional on each draw, we compute the implied historical decomposition. This procedure generates a distribution of time-varying contributions for each shock category.

Figures I.4 and I.5 illustrate the contribution of each structural shock category to the trade-balance-to-GDP ratio and the real exchange rate over time, together with the associated estimation uncertainty. The figures report time-varying posterior contributions along with uncertainty bands based on posterior deciles.

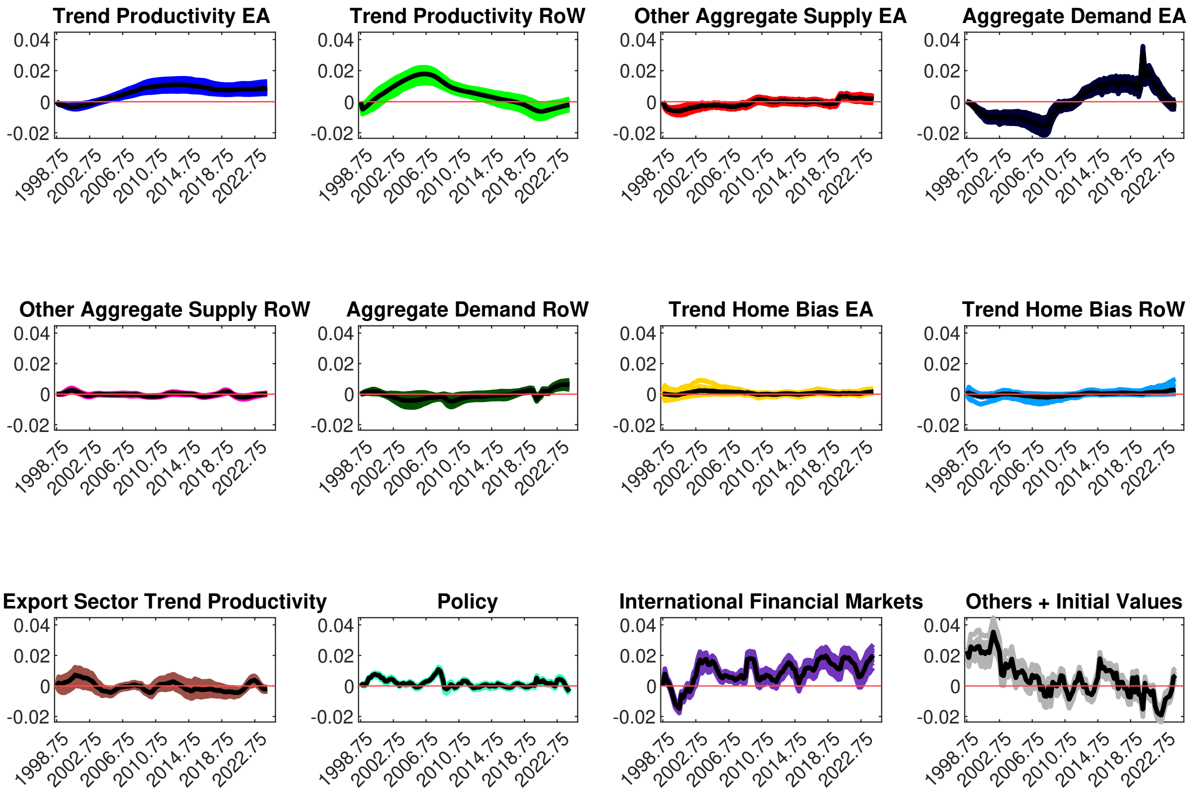


Figure I.4: Posterior decomposition of the trade-balance-to-GDP ratio

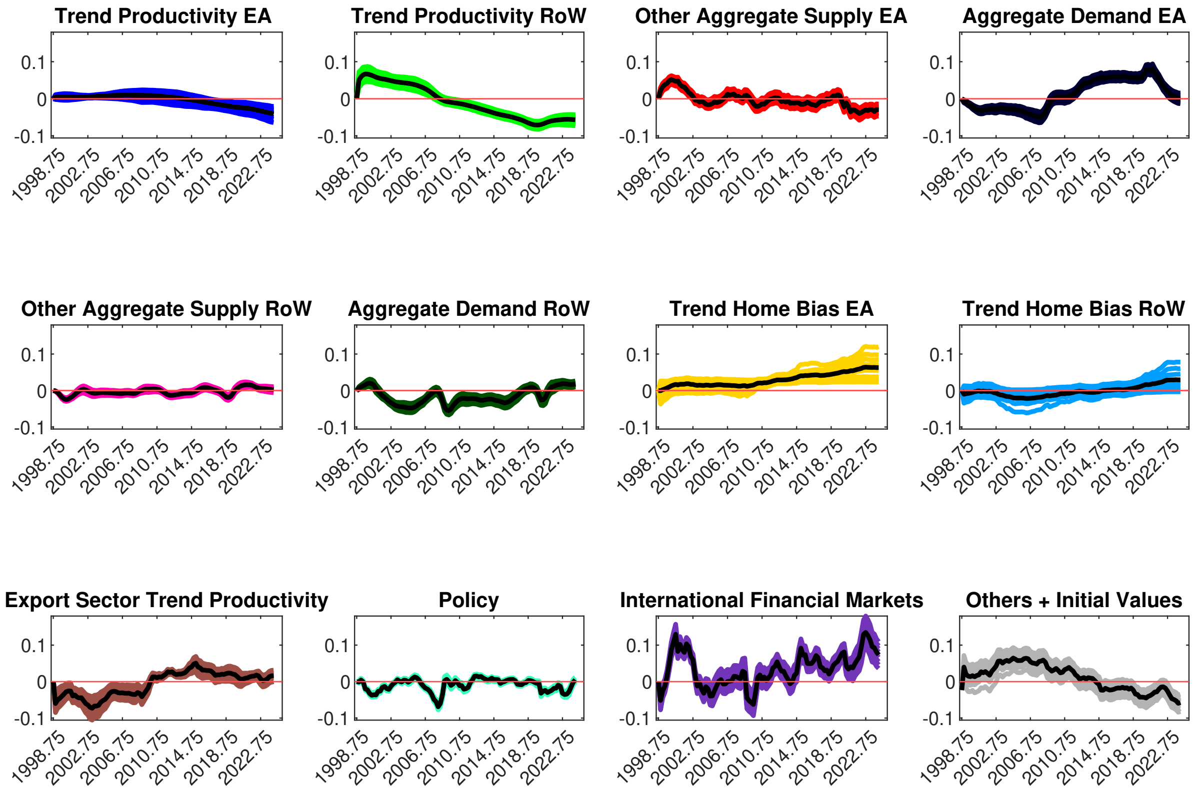


Figure I.5: Posterior decomposition of the real exchange rate

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