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TECHNICAL APPENDIX FOR:  
INTERNATIONAL PORTFOLIOS, CAPITAL ACCUMULATION  
AND FOREIGN ASSETS DYNAMICS

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## A.1. Verifying that the period-by-period household budget constraint (12) is satisfied and that the current account is zero, to first order

In deriving the zero-order equilibrium portfolio  $(S, b)$ , we replaced the period-by-period household budget constraint (12) by the 'static' constraint (26). We now show that when a first-order approximation of the 'static' constraint holds at all dates, then a first-order approximation of (12) holds likewise. Thus it is sufficient to consider the 'static' constraint (26) when solving for  $(S, b)$ .

Following Devereux and Sutherland (2006a,b), we express the period  $t$  budget constraint (12) of country  $i$  as

$$\begin{aligned}
NFA_{i,t+1} &= NX_{i,t} + NFA_{i,t}R_t^{b,i} + \xi_{i,t}, \quad \text{with } NFA_{i,t+1} \equiv p_{j,t}^S S_{j,t+1}^i - p_{i,t}^S S_{i,t+1}^j + p_{i,t}^b b_{i,t+1}^i + p_{j,t}^b b_{j,t+1}^i, \quad j \neq i, \\
NX_{i,t} &\equiv p_{i,t} y_{i,t} - P_{i,t} C_{i,t} - P_{i,t}^I I_{i,t}, \\
\xi_{i,t} &\equiv S_{j,t}^i p_{j,t-1}^S (R_{j,t}^S - R_{i,t}^b) - S_{i,t}^j p_{i,t-1}^S (R_{i,t}^S - R_{i,t}^b) + b_{j,t}^i p_{j,t-1}^b (R_{j,t}^b - R_{i,t}^b), \quad j \neq i.
\end{aligned}$$

$NFA_{i,t+1}$  are country  $i$ 's net foreign assets at the end of period  $t$ , and  $NX_{i,t}$  are  $i$ 's net exports.  $R_{i,t}^S, R_{j,t}^S, R_{i,t}^b, R_{j,t}^b$  are gross equity/bond returns between  $t-1$  and  $t$  (see (15)).  $\xi_{i,t}$  is the "excess return" on the country's net foreign assets (between  $t-1$  and  $t$ ) relative to the return on the good- $i$  bond.<sup>39</sup>

As before, variables without time indices represent (deterministic) steady state values, and  $\widehat{z}_{i,t} \equiv (z_{i,t} - z_i)/z_i$ . Note that  $NFA_i = 0$ ,  $NX_i = 0$ ,  $p^S \equiv p_H^S = p_F^S$ ,  $p^b \equiv p_H^b = p_F^b$ ,  $d \equiv d_H = d_F$ ,  $p \equiv p_H = p_F$ , due to the symmetric structure of the two countries; also,  $R_H^S = R_F^S = R_H^b = R_F^b = 1/\beta$ . A linear approximation of (??) around the steady state yields thus:

$$NFA_{i,t+1} = NX_{i,t} + NFA_{i,t}/\beta + S_j^i p^S \frac{1}{\beta} (\widehat{R}_{j,t}^S - \widehat{R}_{i,t}^b) - S_i^j p^S \frac{1}{\beta} (\widehat{R}_{i,t}^S - \widehat{R}_{i,t}^b) + b_j^i p^b \frac{1}{\beta} (\widehat{R}_{j,t}^b - \widehat{R}_{i,t}^b), \quad j \neq i \quad (41)$$

where  $S_j^i, S_i^j, b_i^i$  and  $b_j^i$  are zero-order portfolio holdings. Symmetry implies  $S_i^j = S_j^i = 1 - S$ ;  $b = b_i^i = -b_j^i$ , for  $j \neq i$ . Hence:

$$NFA_{i,t+1} = NX_{i,t} + NFA_{i,t}/\beta + (S-1)p^S \frac{1}{\beta} (\widehat{R}_{i,t}^S - \widehat{R}_{j,t}^b) + bp^b \frac{1}{\beta} (\widehat{R}_{i,t}^b - \widehat{R}_{j,t}^b), \quad j \neq i. \quad (42)$$

Solving the Euler equations (14) forward gives  $p_{i,t}^S = E_t \sum_{\tau \geq 1} \varrho_{t,t+\tau}^i d_{i,t+\tau}$  and  $p_{i,t}^b = E_t \sum_{\tau \geq 1} \varrho_{t,t+\tau}^i p_{i,t+\tau}$ . Up to first order, the relative stock and bond prices and returns obey thus:

$$\widehat{p}_{i,t}^S - \widehat{p}_{j,t}^S = \frac{1-\beta}{\beta} E_t \sum_{\tau \geq 1} \beta^\tau (\widehat{d}_{i,t+\tau} - \widehat{d}_{j,t+\tau}), \quad \widehat{p}_{i,t}^b - \widehat{p}_{j,t}^b = \frac{1-\beta}{\beta} E_t \sum_{\tau \geq 1} \beta^\tau (\widehat{p}_{i,t+\tau} - \widehat{p}_{j,t+\tau}), \quad j \neq i, \quad (43)$$

$$\widehat{R}_{i,t}^S - \widehat{R}_{j,t}^S = (1-\beta) \widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\widehat{d}_{i,t+\tau} - \widehat{d}_{j,t+\tau}), \quad \widehat{R}_{i,t}^b - \widehat{R}_{j,t}^b = (1-\beta) \widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\widehat{p}_{i,t+\tau} - \widehat{p}_{j,t+\tau}), \quad j \neq i. \quad (44)$$

<sup>39</sup>Note that  $\xi_{i,t} = S_{j,t}^i (d_{j,t} + p_{j,t}^S) - S_{i,t}^j (d_{i,t} + p_{i,t}^S) + (p_{i,t} + p_{i,t}^b) b_{i,t}^i + (p_{j,t} + p_{j,t}^b) b_{j,t}^i - NFA_{i,t} R_{i,t}^b$ . Thus,  $\xi_{i,t}$  is the difference between country  $i$ 's net external wealth (including net dividend and coupon income) at the beginning of period  $t$ , minus the hypothetical value of  $i$ 's net external wealth at the beginning of  $t$  that would obtain if  $i$  fully invested her net external wealth at the end of  $t-1$  in the good- $i$  bond.

with  $\widetilde{E}_t z \equiv E_t z - E_{t-1} z$  (revision of expectation between  $t - 1$  and  $t$ ). Thus,  $E_t(\widehat{R_{i,t+\tau}^S} - \widehat{R_{j,t+\tau}^S}) = E_t(\widehat{R_{i,t+\tau}^b} - \widehat{R_{j,t+\tau}^b}) = 0$  for  $\tau > 0$ : up to first order, the expected value of future excess returns is zero.

Solving (42) forward (imposing the no-Ponzi/transversality condition  $\lim_{\tau \rightarrow \infty} E_t \beta^\tau NFA_{i,t+\tau} = 0$ ) gives the following present value budget constraint:

$$E_t \sum_{\tau \geq 0} \beta^\tau (-NX_{i,t+\tau}) = NFA_{i,t}/\beta + (S-1)d\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\widehat{d_{i,t+\tau}} - \widehat{d_{j,t+\tau}}) + bp\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\widehat{p_{i,t+\tau}} - \widehat{p_{j,t+\tau}}), \quad j \neq i, \quad (45)$$

where we used that fact that  $p^S = d\beta/(1 - \beta)$ ,  $p^b = p\beta/(1 - \beta)$ .

(45) holds if and only if:

$$\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (-NX_{i,t+\tau}) = (S-1)d\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\widehat{d_{i,t+\tau}} - \widehat{d_{j,t+\tau}}) + bp\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\widehat{p_{i,t+\tau}} - \widehat{p_{j,t+\tau}}), \quad j \neq i \quad (46)$$

$$\text{and} \quad E_{t-1} \sum_{\tau \geq 0} \beta^\tau (-NX_{i,t+\tau}) = NFA_{i,t}/\beta. \quad (47)$$

(46) shows that, up to first order, date  $t$  innovations to the expected present value of current and future country  $i$  net imports have to equal innovations to the present value of net dividend and net bond income received by country  $i$ .

### The 'static' budget constraint

In deriving the zero-order equilibrium portfolio, we replaced the period-by-period household budget constraint (12) by the 'static' budget constraint:  $P_{i,t}C_{i,t} = w_{i,t}l_{i,t} + Sd_{i,t} + (1-S)d_{j,t} + b(p_{i,t} - p_{j,t})$  (see (26)). This constraint can be expressed as:  $-NX_{i,t} = (S-1)(d_{i,t} - d_{j,t}) + b(p_{i,t} - p_{j,t})$ .<sup>40</sup> Equivalently:

$$-NX_{i,t} = (S-1)d(\widehat{d_{i,t}} - \widehat{d_{j,t}}) + bp(\widehat{p_{i,t}} - \widehat{p_{j,t}}), \quad j \neq i. \quad (48)$$

It is clear that when (48) holds at all dates, then the present value budget constraint (46) is also satisfied. We show next that (47) entails a restriction on the first-order (time-varying) *deviations* of portfolio holdings from zero-order portfolio holdings. This implies that, when solving for the *zero-order* portfolio zero-order portfolio  $(S, b)$ , it is sufficient to consider the 'static' budget constraint (26).

### A restriction on first-order accurate (time-varying) portfolio holdings

<sup>40</sup>Subtracting  $w_{i,t}l_{i,t}$  and  $d_{i,t}$  from both sides of the static constraint gives:  $P_{i,t}C_{i,t} - w_{i,t}l_{i,t} - d_{i,t} = (S-1)(d_{i,t} - d_{j,t}) + b(p_{i,t} - p_{j,t})$ . The left-hand side of this expression equals  $-NX_{i,t}$  (as  $d_{i,t} \equiv p_{i,t}y_{i,t} - w_{i,t}l_{i,t} - P_{i,t}^I I_{i,t}$ ).

Substitution of (48) into (47) yields:

$$NFA_{i,t} = (S-1)\beta E_{t-1} \sum_{\tau \geq 0} \beta^\tau d(\widehat{d_{i,t+\tau}} - \widehat{d_{j,t+\tau}}) + b\beta E_{t-1} \sum_{\tau \geq 0} \beta^\tau p(\widehat{p_{i,t+\tau}} - \widehat{p_{j,t+\tau}}), \quad j \neq i. \quad (49)$$

Using the formulae for relative asset prices (43), we can write this as:

$$NFA_{i,t} = (S-1)p^S(\widehat{p_{i,t-1}^S} - \widehat{p_{j,t-1}^S}) + bp^b(\widehat{p_{i,t-1}^b} - \widehat{p_{j,t-1}^b}), \quad j \neq i. \quad (50)$$

Linearizing the expression  $NFA_{i,t} = p_{j,t-1}^S S_{j,t}^i - p_{i,t-1}^S S_{i,t}^j + p_{i,t-1}^b b_{i,t}^i + p_{j,t-1}^b b_{j,t}^i$  gives

$$NFA_{i,t} \equiv (S-1)p^S(\widehat{p_{i,t-1}^S} - \widehat{p_{j,t-1}^S}) + bp^b(\widehat{p_{i,t-1}^b} - \widehat{p_{j,t-1}^b}) + (\nabla S_{j,t}^i - \nabla S_{i,t}^j)p^S + (\nabla b_{i,t}^i + \nabla b_{j,t}^i)p^b, \quad j \neq i, \quad (51)$$

where  $\nabla S_{j,t}^i \equiv S_{j,t}^i - (1-S)$ ,  $\nabla S_{i,t}^j \equiv S_{i,t}^j - (1-S)$ ,  $\nabla b_{i,t}^i \equiv b_{i,t}^i - b$ ,  $\nabla b_{j,t}^i \equiv b_{j,t}^i - (-b)$  denote the deviations of portfolio holdings at the end of period  $t-1$  from the zero-order portfolio. (50) and (51) imply that, to first order, the value of country' net external assets, evaluated at steady state asset prices is zero:

$$(\nabla S_{j,t}^i - \nabla S_{i,t}^j)p^S + (\nabla b_{i,t}^i + \nabla b_{j,t}^i)p^b = (S_{j,t}^i - S_{i,t}^j)p^S + (b_{i,t}^i + b_{j,t}^i)p^b = 0, \quad j \neq i. \quad (52)$$

### The current account

The period  $t$  current account of country  $i$  is:  $CA_{i,t} = (S_{j,t+1}^i - S_{j,t}^i)p_{j,t}^S - (S_{i,t+1}^j - S_{i,t}^j)p_{i,t}^S + (b_{i,t+1}^i - b_{i,t}^i)p_{i,t}^b + (b_{j,t+1}^i - b_{j,t}^i)p_{i,t}^b$ . Linearization of this expression gives:  $CA_{i,t} = \{(S_{j,t+1}^i - S_{j,t}^i) - (S_{i,t+1}^j - S_{i,t}^j)\}p^S + (b_{i,t+1}^i - b_{i,t}^i + b_{j,t+1}^i - b_{j,t}^i)p^b$ . It thus follows from (52) that  $CA_{i,t} = 0$ , up to first order.

## A.2. Returns and the equilibrium portfolio

Equation (37) in the main text shows that the zero-order local equity position  $S$  depends on the covariance between components of relative (Home vs. Foreign) wage incomes and dividend payments that are orthogonal to the terms of trade:  $S = \frac{1}{2} - \frac{1-\kappa}{2} \frac{1-\kappa}{\kappa-\Lambda} \frac{Cov_{\hat{q}}(w_{i,t}, \hat{d}_t)}{Var_{\hat{q}}(\hat{d}_t)}$ . We now show that  $S$  can equivalently be expressed as a function of the covariance between components of relative (Home vs. Foreign) human capital returns and equity returns that are orthogonal to the return differential between the Home-good and Foreign-good bonds.

As shown above, country  $i$  net imports can be expressed as:  $-NX_{i,t} = P_{i,t}C_{i,t} - w_{i,t}l_{i,t} - d_{i,t}$ ; this can be written as:  $-NX_{i,t} = py_i(1-\Lambda)\widehat{P_{i,t}C_{i,t}} - (1-\kappa)py_i\widehat{w_{i,t}l_{i,t}} - py_i(\kappa-\Lambda)\widehat{d_{i,t}}$ .<sup>41</sup> Inserting this

<sup>41</sup>NB Steady state consumption spending is a fraction  $(1-\Lambda)$  of output, where  $\Lambda$  is the steady state ratio of investment spending to GDP; wage income and dividends account for fractions  $1-\kappa$  and  $\kappa-\Lambda$  of output, respectively.

expression into (46) gives:

$$(1-\Lambda)\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau P_{i,t+\tau} \widehat{C}_{i,t+\tau} = (1-\kappa)\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau w_{i,t+\tau} \widehat{l}_{i,t+\tau} + S(\kappa-\Lambda)\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau \widehat{d}_{i,t+\tau} \\ + (1-S)(\kappa-\Lambda)\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau \widehat{d}_{j,t+\tau} + \widetilde{b}\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau (\widehat{p}_{i,t+\tau} - \widehat{p}_{j,t+\tau}), \quad j \neq i. \quad (53)$$

where  $\widetilde{b} \equiv b/y_i$  is the local-good bond holding divided by steady state output.

Subtracting the linearized present value budget constraint (53) of country F from that of country H yields:

$$(1-\Lambda)\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau [P_{H,t+\tau} \widehat{C}_{H,t+\tau} - P_{F,t+\tau} \widehat{C}_{F,t+\tau}] = (1-\kappa)\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau [w_{H,t+\tau} \widehat{l}_{H,t+\tau} - w_{F,t+\tau} \widehat{l}_{F,t+\tau}] + \\ (2S-1)(\kappa-\Lambda)\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau [\widehat{d}_{H,t+\tau} - \widehat{d}_{F,t+\tau}] + 2\widetilde{b}\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau \widehat{q}_{t+\tau}, \quad (54)$$

where  $q_t \equiv p_{H,t}/p_{F,t}$  are the Home terms of trade. Effective market completeness (up to first order) implies that  $P_{H,t} \widehat{C}_{H,t} - P_{F,t} \widehat{C}_{F,t} = (1 - \frac{1}{\sigma})q_t$  (see (28)). Thus, innovations to the present value of relative consumption spending are perfectly correlated with the return differential between Home-good and Foreign-good bonds (from (44)):

$$\widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau [P_{H,t+\tau} \widehat{C}_{H,t+\tau} - P_{F,t+\tau} \widehat{C}_{F,t+\tau}] = (1 - \frac{1}{\sigma}) \frac{1}{1-\beta} (R_{H,t}^b - R_{F,t}^b).$$

Define the return on country  $i$  Human capital as:  $R_{i,t}^W \equiv \frac{P_{i,t}^W + w_{i,t} l_{i,t}}{P_{i,t}^W}$ , where  $P_{i,t}^W \equiv E_t \sum_{\tau \geq 1} \beta^\tau w_{i,t+\tau} l_{i,t+\tau}$  is the present value of the country  $i$  labor income; linearizing these formulae gives:

$$\widehat{R}_{H,t}^W - \widehat{R}_{F,t}^W = (1-\beta) \widetilde{E}_t \sum_{\tau \geq 0} \beta^\tau [w_{H,t+\tau} \widehat{l}_{H,t+\tau} - w_{F,t+\tau} \widehat{l}_{F,t+\tau}].$$

Using the expression for the cross-country equity return differential shown in (44), we can thus express (54) as:

$$(1-\Lambda)(1 - \frac{1}{\sigma})R_t^b = (1-\kappa)R_t^W + (2S-1)(\kappa-\Lambda)R_t^S + 2\widetilde{b}R_t^b.$$

where  $R_t^b \equiv R_{H,t}^b - R_{F,t}^b$ ,  $R_t^W \equiv R_{H,t}^W - R_{F,t}^W$ ,  $R_t^S \equiv R_{H,t}^S - R_{F,t}^S$  are Home-Foreign return differentials for bonds, Human capital and equity, respectively. This condition implies:

$$S = \frac{1}{2} - \frac{1}{2} \frac{1-\kappa}{\kappa-\Lambda} \frac{Cov_{R^b}(R_t^W, R_t^S)}{Var_{R^b}(R_t^S)}, \quad (55)$$

with  $Cov_{R^b}(R_t^W, R_t^S) \equiv E\{R_t^W - P[R_t^W|R_t^b]\{R_t^S - P[R_t^S|R_t^b]\}$ ,  $Var_{R^b}(R_t^S) \equiv E\{R_t^S - P[R_t^S|R_t^b]\}^2$ , where  $P[R_t^W|R_t^b]$  is the linear projection of  $R_t^W$  on  $R_t^b$ . Thus, the local equity share can be expressed as a function of the covariance between the components of relative (Home vs. Foreign) human capital returns and (relative) equity returns that are orthogonal to (relative) bond returns: equity home bias arises when that covariance is negative.

The model here generates a negative covariance. In the main text we showed that a combination of exogenous shocks that raises relative Home real investment spending, without affecting the terms of trade has these consequences: relative Home wage income rises, and the relative dividend of the Home firm falls. The same logic also applies directly to capitalized income streams, and thus to returns. Consider a combination of exogenous innovations that raises the present discounted value of relative Home real investment spending, without changing the present value of (current and future) Home terms of trade; that combination of shocks raises the present value of relative Home wage income, while lowering the present value of relative Home dividends; in other terms, such a combination of shocks has no effect on the return differential between Home-good and Foreign-good bonds, and no effect on the present value of efficient relative Home consumption spending; however, it raises the relative return on Home human capital, while reducing the relative return of the Home stock. Holding constant the bond return differential, the relative return on Home human capital co-moves thus negatively with the relative Home stock return:  $Cov_{R^b}(R_t^W, R_t^S) < 0$ .

### A.3. Corporate Debt

The following results hold when firms (partly) finance investment spending by issuing debt: (1) The structure of equilibrium equity portfolio is unaffected (i.e. the equity portfolio continues to be given by (32)); (2) Households's holdings of corporate debt exhibit a home bias in the same proportion as for stocks; (3) The net external bond positions are altered; if firms issue debt denominated in their local good, the range of parameters increases for which a country's net local-good debt position is negative.

To understand these results note that, in the economy here, the Modigliani-Miller theorem holds, i.e. the issuance of corporate debt does not affect firm values or equilibrium consumptions. Assume that firms issue bonds denominated in the good that they produce, and that one unit of a corporate bond pays one unit of output in all future periods; assume also that a constant share  $\mu$  of the *net* investment of the country  $i$  firm  $P_{i,t}^I (I_{i,t} - \delta K_{i,t})$  is financed by issuing debt.<sup>42</sup> The rest of investment spending  $P_{i,t}^I I_{i,t} - \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t})$  is financed through retained earnings.

Let  $D_{it}$  denote the outstanding debt of the country  $i$  firm, at the beginning of period  $t$ . The price of one unit of the debt is  $p_{it}^b$ . Suppose that the country  $i$  household holds a fraction  $S$   $[1 - S]$  of local [foreign] equity and of the outstanding local [foreign] corporate bonds.

This portfolio strategy allows the household to off-set the implicit debt position entailed by its equity position. When  $S$  is set at the optimal value in the baseline model (where firms are fully equity financed,  $\mu = 0$ ), such a portfolio strategy thus generates the same financial income as the household's portfolio in the baseline model; hence that portfolio strategy replicates the efficient consumption allocation (up to first order). To see this, note that in the present setting the period  $t$  dividend of the country  $i$  firm equals a share  $\kappa$  of its output,  $\kappa p_{i,t} y_{i,t}$  (NB  $\kappa$  is the capital share) minus retained earnings  $P_{i,t}^I I_{i,t} - \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t})$ , less the coupon payment  $p_{it} D_{it}$  made by the firm (to holders of its bonds).

Thus, the date  $t$  dividend is:

$$d_{it}(\mu) \equiv \kappa p_{i,t} y_{i,t} - [P_{i,t}^I I_{i,t} - \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t})] - p_{it} D_{it}$$

The firm's issuance of new corporate bonds in period  $t$  is given by:  $p_{it}^D (D_{it+1} - D_{it}) = \mu P_{i,t}^I (I_{i,t} - \delta K_{i,t})$ . When the country  $i$  household holds a share  $S$  of local equity and local corporate debt, then that household derives the following income from local equity and corporate debt, in period  $t$  (net of the amount spent

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<sup>42</sup>Note that results (1) and (2) do not hinge on these assumptions.

at  $t$  to purchase a  $S$  fraction of the newly issued local corporate debt):

$$Sd_{it}(\mu) + Sp_{it}D_{it} - Sp_{it}^D(D_{it+1} - D_{it}) = S(\kappa p_{i,t}y_{i,t} - P_{i,t}^I I_{i,t}) = Sd_{it}(\mu = 0).$$

This corresponds to the dividend income of the household, from her holdings of local equity, in the baseline model in which firms do not issue debt  $d_{it}(\mu = 0)$ . By the same reasoning, the portfolio strategy described above ensures that the household receives an income from her holdings of foreign equity and foreign corporate debt that equals her dividend income from foreign equity, in the baseline model. In order to replicate efficient risk-sharing (up to first order), the household in addition has to hold the *same* amount of local-good and foreign-good *non-corporate* debt as in the baseline model.

In summary, the equilibrium holdings of local and foreign equity shares and of non-corporate bonds are the same as in the baseline model, but investors now also hold a share  $S$  of domestic corporate debt and a share  $1 - S$  of foreign corporate debt. When the country  $i$  firm issues one unit of debt denominated in local good  $i$ , then the country's overall (household+firm) net local good debt position changes by  $1 - S < 0$  units, as a share  $S$  of the new debt is purchased by the local household (while a share  $1 - S$  of the new debt is bought by the foreign household). Thus the presence of *local-good* corporate debt lowers the country's overall net local-good debt. When *all* corporate debt is denominated in the local good, the country's overall (household+corporate) local-good debt position is  $b + (S - 1)D_{Ht} < b$ , where  $b$  is the local household's holding of local-good non-corporate debt (as discussed above,  $b$  has the same value as in the baseline model without corporate debt).