



ELSEVIER

Journal of Monetary Economics 49 (2002) 989–1015

Journal of  
MONETARY  
ECONOMICS

www.elsevier.com/locate/econbase

# Monetary policy rules in the open economy: effects on welfare and business cycles<sup>☆</sup>

Robert Kollmann<sup>a,b,\*</sup>

<sup>a</sup> *Department of Economics, University of Bonn, 24-42 Adenauerallee, D-53113 Bonn, Germany*

<sup>b</sup> *Centre for Economic Policy Research, UK*

Received 16 November 2001; received in revised form 4 March 2002; accepted 5 March 2002

---

## Abstract

This paper computes welfare maximizing Taylor-style interest rate rules, in a business cycle model of a small open economy. The model assumes staggered price setting and shocks to domestic productivity, to the world interest rate, to world inflation, and to the uncovered interest rate parity condition. Optimized policy rules have a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility. The country responds to an increase in external volatility by holding more foreign assets. The policy rule affects the variance and the mean of consumption. The effect on the mean matters significantly for welfare. © 2002 Elsevier Science B.V. All rights reserved.

*JEL classification:* E4; F3; F4

*Keywords:* Open economy; Interest rate rules; Business cycles

---

---

<sup>☆</sup>This paper was prepared for the November 2001 Carnegie-Rochester Conference on Public Policy. I thank Ben McCallum for encouragement and advice, and Andy Levin and other conference participants for useful comments. I am grateful to Jinill Kim, Stephanie Schmitt-Grohé, and Chris Sims, for discussions/advice on computational issues. Thanks for helpful discussions are also due to Gianluca Benigno, Pierpaolo Benigno, Paul Bergin, Luca Dedola, Chris Erceg, Mark Gertler, Soyoung Kim, Sylvain Leduc, Tommaso Monacelli, Ed Nelson, Paolo Pesenti, Frank Smets, Pedro Teles, Harald Uhlig, Martín Uribe, Michael Woodford, and Raf Wouters, as well as to seminar participants at the North American Winter Meetings of the Econometric Society.

\*Corresponding author. Tel.: +49-228-734073; fax: +49-228-739100.

*E-mail address:* kollmann@wiwi.uni-bonn.de (R. Kollmann).

## 1. Introduction

The effect of the monetary policy regime on welfare and business cycles is a key question in economics. This paper examines that question using a micro-based quantitative (calibrated) business cycle model of a small open economy in which monetary policy affects real activity because of staggered price setting.

Much effort has recently been devoted to develop dynamic general equilibrium models of open economies with monopolistic competition and sluggish prices (or wages)—see Lane (2001) for a survey of that work, often referred to as “New Open Economy Macroeconomics” (NOEM). An important strand of the NOEM literature uses highly stylized models (for which analytical results can be worked out) to determine welfare under alternative exchange rate regimes and to derive optimal monetary policy rules. The simplifying assumptions generally made in these models include, in particular: full international risk sharing, a stripped-down structure of shocks (mostly just one type of shock—productivity shocks), and the absence of physical capital.<sup>1</sup> Another strand of the literature develops quantitative business cycle models that can be used to study the key features of international macroeconomic data.<sup>2</sup>

The models studied in the first strand seem too stylized for empirical analysis, whereas computing welfare (and welfare maximizing policy rules) in quantitative business cycle models has, until now, not been practically feasible, given available numerical techniques. The paper here bridges these two approaches by determining welfare maximizing Taylor (1993a)-style interest rate rules, using a quantitative business cycle model. This is made possible by recent advances in solving dynamic models (Sims, 2000).

The model here extends the sticky-prices open economy model that Kollmann (2001a) calibrated to data for Japan, Germany and the U.K. It assumes imperfect international risk sharing due to incomplete international financial markets (transactions restricted to trade in bonds) and physical capital (like standard business cycle models). In the model, there are shocks to domestic productivity, to the world interest rate, to world inflation, and to the uncovered interest parity condition (“UIP shocks”). Monetary policy is described by a rule according to which the nominal interest rate is set as a function of the inflation rate and of GDP.

---

<sup>1</sup>See, for example, Bacchetta and van Wincoop (2000), Benigno (2000, 2001), Clarida et al. (2001), Corsetti and Pesenti (2001), Devereux and Engel (2000), Galí and Monacelli (2000), Obstfeld and Rogoff (2000), Parrado and Velasco (2001), and Sutherland (2001).

<sup>2</sup>See, for example, Batini et al. (2001), Benigno (1999), Bergin (2001), Betts and Devereux (2001), Chari et al. (2000), Collard and Dellas (2002), Dedola and Leduc (2001), Duarte and Stockman (2001), Erceg and Levin (2001), Faia (2001), Ghironi and Rebucci (2001), Hairault et al. (2001), Kollmann (2001a, b; 2002), McCallum and Nelson (1999, 2000), Monacelli (1999), Schmitt-Grohé and Uribe (2001a), and Smets and Wouters (2000, 2001).

Imperfect risk sharing is more realistic than the complete risk sharing assumed in previous welfare analyses.<sup>3</sup> In the bonds-only structure here, macroeconomic variability affects the mean net foreign asset position—which has significant consequences for welfare.<sup>4</sup> That effect is not present in models with complete risk sharing.

UIP shocks are assumed here because of the well-documented strong and persistent departures from UIP during the post-Bretton Woods (post-BW) era (e.g., Lewis, 1995). Also, econometric attempts to explain short-run exchange rate movements from changes in monetary policy (and other macro fundamentals) have failed (e.g., Rogoff, 2000). Structural models driven only by “traditional” fundamentals generate predicted exchange rate variability that is much smaller than that seen in post-BW data; thus, such models are not well suited to analyzing a floating-rate regime. The model here—with UIP shocks—generates more realistic exchange rate volatility; the predicted standard deviation of the Hodrick–Prescott (HP) filtered nominal exchange rate is 7.1% (in a version of the model without UIP shocks, the corresponding standard deviation is 3.4%); the standard deviations of HP filtered quarterly exchange rates of Japan, Germany and the U.K. vis-à-vis the U.S. were about 9% during the post-BW era. (In the NOEM literature, only McCallum and Nelson (1999) and Batini and Nelson (2000) compare alternative policy rules using models with UIP shocks, but these authors do not compute welfare.)

The present model is solved using Sims’ (2000) new method that is based on a quadratic approximation of the equilibrium conditions. In contrast to the solution methods based on linear approximations that are widely used in macroeconomics, the Sims approach allows to compute the (second-order accurate) effect of the policy rule on expected values of macroeconomic variables—an effect that is crucial for welfare in the model here. Compared with other non-linear solution methods (see Judd, 1998), the Sims method has two key advantages—the ease with which it can be applied to models with a large number of state variables and its high computational speed. These features allow to numerically determine the coefficients of the monetary policy rule that maximize welfare.<sup>5</sup>

The optimized rule entails rather strict (but not perfect) targeting of the growth rate of the domestic producer price index (PPI): the implied standard deviation of PPI inflation is low (0.08%). It yields a welfare level that is close to that of the economy under flexible prices. The domestic interest rate falls in response to positive shocks to domestic productivity; it shows little response to UIP shocks and to shocks to the world interest rate and to world inflation. The rule implies significant nominal

---

<sup>3</sup> Models with complete risk sharing typically generate cross-country consumption correlations that are much too high, when compared to the data; a bonds-only structure can generate correlations that are markedly lower (and, thus, closer to the data); e.g., Backus et al. (1995) and Kollmann (1995, 1996).

<sup>4</sup> In the model, stationarity of the net asset position is ensured by assuming a debt-elastic interest rate premium on international bonds.

<sup>5</sup> Smets and Wouters (2000, 2001) also discuss welfare in a calibrated open economy model with incomplete financial markets (but without capital or UIP shocks). These authors do not compute the effect of the policy rule on expected values of macrovariables.

and real exchange rate volatility. Permitting a direct response of the interest rate rule to the nominal exchange rate yields only a minuscule welfare gain. Under the optimized rule, productivity shocks are the main source of fluctuations in output and consumption, while UIP shocks are the dominant source of exchange rate fluctuations. UIP shocks have a positive effect on welfare i.a. because they lead the country to hold a larger stock of foreign bonds. Hence, with UIP shocks, the country is wealthier (on average), and it enjoys higher mean consumption.

Prior research shows that when price stickiness (in producer currency) is the only economic distortion (so that the flex-prices equilibrium of the economy is efficient), and exchange rate changes are fully and immediately passed through into import prices (ensuring that the law of one price (LOP) holds), then welfare maximizing monetary policy requires perfect stabilization of the domestic PPI (e.g., Aoki, 2001; Devereux and Engel, 2000; Galí and Monacelli, 2000). That policy replicates the flex-prices equilibrium and entails a floating exchange rate. Full PPI stabilization is not optimal when (as in the model here) the flex-prices equilibrium is not efficient (here: i.a. monopolistic distortions) or when exchange rate pass through is limited. It thus seems worth noting that the optimized policy rule, in the economy discussed here, does entail rather strict (but not perfect) PPI inflation targeting, and that it yields a welfare level close to that in the flex-prices economy. The results suggest that (near) PPI inflation stabilization is also desirable under the more realistic assumption that exchange rate pass through is limited, as a result of pricing-to-market (PTM) (the data clearly reject full pass through and the LOP; see, e.g., Knetter, 1993; Campa and Goldberg, 2001).

In the model here, pegging the exchange rate reduces welfare. Under a peg, external shocks require strong and immediate adjustment of the domestic interest rate—these shocks thus have a more destabilizing effect on consumption (than under the optimized rule). In addition, a peg reduces mean consumption, since the increased volatility of goods demand under a peg induces firms to set higher price-marginal cost markups. Under the plausible assumption that pegging the exchange rate reduces the variance of UIP shocks (UIP shocks were smaller under the BW system than in the post-BW era), the country holds a smaller stock of foreign bonds under a peg—which also lowers welfare.

The model captures the fact that nominal and real exchange rate volatility among the major currency blocs has risen sharply after the end of the BW system, whereas output volatility has shown little change (e.g., Baxter and Stockman, 1989).

Section 2 of this paper presents the model and discusses the solution method, Section 3 presents the results and Section 4 concludes.

## **2. The model**

A small open economy with a representative household, firms, and a central bank is considered (the structure of preferences and technologies follows Kollmann, 2001a). The economy produces a single non-tradable final good and a continuum of tradable intermediate goods indexed by  $s \in [0, 1]$ . It imports a continuum of foreign

intermediate goods, also indexed by  $s \in [0, 1]$ . Domestic and foreign intermediate goods are used by perfectly competitive firms to produce the final good which is consumed and used for investment. There is monopolistic competition in intermediate goods markets—each intermediate good is produced or imported by a single firm. Intermediate goods producers use domestic capital and labor as inputs—capital and labor are immobile internationally. The household owns all domestic producers and the capital stock, which it rents to producers. It also supplies labor. The markets for rental capital and for labor are competitive.

2.1. Final good production

The final good is produced using the aggregate technology

$$Z_t = \{(\alpha^d)^{1/\vartheta}(Q_t^d)^{(\vartheta-1)/\vartheta} + (\alpha^m)^{1/\vartheta}(Q_t^m)^{(\vartheta-1)/\vartheta}\}^{\vartheta/(\vartheta-1)}, \tag{1}$$

with  $\alpha^d, \alpha^m > 0, \alpha^d + \alpha^m = 1, \vartheta > 0$ .  $Z_t$  is final good output at date  $t$ ;  $Q_t^d, Q_t^m$  are quantity indices of domestic and imported intermediate goods, respectively:  $Q_t^i = \{\int_0^1 q_t^i(s)^{(v-1)/v} ds\}^{v/(v-1)}$  with  $v > 1$  for  $i = d, m$ , where  $q_t^d(s)$  and  $q_t^m(s)$  are quantities of the domestic and imported type  $s$  intermediate goods. Let  $p_t^d(s)$  and  $p_t^m(s)$  be the prices of these goods in domestic currency. Cost minimization in final good production implies

$$q_t^i(s) = (p_t^i(s)/P_t^i)^{-v} Q_t^i, \quad Q_t^i = \alpha^i (P_t^i/P_t)^{-\vartheta} Z_t \quad \text{for } i = d, m, \tag{2}$$

with

$$P_t^i = \left\{ \int_0^1 p_t^i(s)^{1-v} ds \right\}^{1/(1-v)}, \quad P_t = \{\alpha^d (P_t^d)^{1-\vartheta} + \alpha^m (P_t^m)^{1-\vartheta}\}^{1/(1-\vartheta)}. \tag{3}$$

$P_t^d [P_t^m]$  is a price index for domestic [imported] intermediate goods that are sold in the domestic market. Perfect competition in the final good market implies that the good’s price is  $P_t$  (its marginal cost is  $\{\alpha^d (P_t^d)^{1-\vartheta} + \alpha^m (P_t^m)^{1-\vartheta}\}^{1/(1-\vartheta)}$ ).

2.2. Intermediate goods firms

The technology of the firm that produces domestic intermediate good  $s$  is

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1. \tag{4}$$

$y_t(s)$  is the firm’s output at date  $t$ ;  $\theta_t$  is an exogenous productivity parameter that is identical for all domestic intermediate goods producers;  $K_t(s)$  and  $L_t(s)$  are the amounts of capital and labor used by the firm.

Let  $R_t$  and  $W_t$  be the rental rate of capital and the wage rate. Cost minimization implies:  $L_t(s)/K_t(s) = \psi^{-1}(1 - \psi)R_t/W_t$ . The firm’s marginal cost is  $MC_t = (1/\theta_t)R_t^\psi W_t^{1-\psi} \psi^{-\psi} (1 - \psi)^{\psi-1}$ .

The firm’s good is sold in the domestic market and exported:  $y_t = q_t^d(s) + q_t^x(s)$ , where  $q_t^d(s) [q_t^x(s)]$  is domestic [export] demand. The export demand function is

assumed to resemble the domestic demand function (2):

$$q_t^x(s) = (p_t^x(s)/P_t^x)^{-\nu} Q_t^x, \quad \text{with } Q_t^x = (P_t^x/P_t^*)^{-\eta}, \quad \eta > 0, \tag{5}$$

where  $p_t^x(s)$  is the firm’s export price in foreign currency, while

$$Q_t^x = \left\{ \int_0^1 q_t^x(s)^{(v-1)/\nu} ds \right\}^{v/(v-1)}, \quad P_t^x = \left\{ \int_0^1 p_t^x(s)^{1-\nu} ds \right\}^{1/(1-\nu)} \tag{6}$$

are a quantity index and a price index for the country’s exports.  $P_t^*$  is the world price level and also represents the purchase price of foreign intermediate goods paid by domestic importers;  $P_t^*$  is exogenous.

The profits of a domestic intermediate good producer,  $\pi_t^{dx}$ , and of an intermediate good importer,  $\pi_t^m$ , are:

$$\begin{aligned} \pi_t^{dx}(p_t^d(s), p_t^x(s)) &= (p_t^d(s) - MC_t)(p_t^d(s)/P_t^d)^{-\nu} Q_t^d + (e_t p_t^x(s) - MC_t)(p_t^x(s)/P_t^x)^{-\nu} Q_t^x, \\ \pi_t^m(p_t^m(s)) &= (p_t^m(s) - e_t P_t^*)(p_t^m(s)/P_t^m)^{-\nu} Q_t^m, \end{aligned} \tag{7}$$

where  $e_t$  is the nominal exchange rate, expressed as the domestic currency price of foreign currency.

Motivated by the empirical failure of the LOP, and in particular by widespread PTM behavior (e.g., Knetter, 1993), it is assumed that intermediate goods producers can price discriminate between the domestic market and the export market ( $p_t^d(s) \neq e_t p_t^x(s)$  is possible), and that they set prices in the currencies of their customers.

There is staggered price setting, à la Calvo (1983): intermediate goods firms cannot change prices, in buyer currency, unless they receive a random “price-change signal”. The probability of receiving this signal in any particular period is  $1 - d$ , a constant. Thus, the mean price-change-interval is  $1/(1 - d)$ . Following Yun (1996) and Erceg et al. (2000) it is assumed that when a firm does not receive a “price-change signal”, its price is automatically increased at the steady state growth factor of the price level (in the buyer’s country). (Throughout this paper, the term “steady state” refers to the deterministic steady state.) Firms are assumed to meet all demand at posted prices.

Consider an intermediate good producer that, at time  $t$ , sets a new price in the domestic market,  $p_{t,t}^d$ . If no “price-change signal” is received between  $t$  and  $t + \tau$ , the price is  $p_{t,t}^d \Pi^\tau$  at  $t + \tau$ , where  $\Pi$  is the steady-state growth factor of the domestic price level. The firm sets

$$p_{t,t}^d = \text{Arg Max}_{\mathfrak{P}} \sum_{\tau=0}^{\tau=\infty} d^\tau E_t \{ \rho_{t,t+\tau} \pi_{t+\tau}^{dx}(\mathfrak{P} \Pi^\tau, p_{t+\tau}^x(s)) / P_{t+\tau} \},$$

where  $\rho_{t,t+\tau}$  is a pricing kernel (for valuing date  $t + \tau$  payoffs) that equals the household’s marginal rate of substitution between consumption at  $t$  and at  $t + \tau$  (see discussion below). Let  $\Xi_{t,t+\tau}^i = \rho_{t,t+\tau} (P_t/P_{t+\tau}) Q_{t+\tau}^i (P_{t+\tau}^i)^{\nu}$ , for  $i = d, x$ . The solution

of the maximization problem regarding  $p_{t,t}^d$  is

$$p_{t,t}^d = (v/(v - 1)) \left\{ \sum_{\tau=0}^{\infty} (d\Pi^{-v})^\tau E_t \Xi_{t,t+\tau}^d MC_{t+\tau} \right\} / \left\{ \sum_{\tau=0}^{\infty} (d\Pi^{1-v})^\tau E_t \Xi_{t,t+\tau}^d \right\}. \tag{8}$$

Analogously, an intermediate good producer that gets to choose a new export price at date  $t$  sets that price at

$$p_{t,t}^x = (v/(v - 1)) \left\{ \sum_{\tau=0}^{\infty} (d(\Pi^*)^{-v})^\tau E_t \Xi_{t,t+\tau}^x MC_{t+\tau} \right\} / \left\{ \sum_{\tau=0}^{\infty} (d(\Pi^*)^{1-v})^\tau E_t \Xi_{t,t+\tau}^x e_{t+\tau} \right\}, \tag{9}$$

where  $\Pi^*$  is the steady-state growth factor of the world price level.

Firms that import foreign intermediate goods are owned by risk-neutral foreigners who discount future payoffs using the world nominal interest rate.<sup>6</sup> An importer that gets to set a new price selects

$$p_{t,t}^m = \text{Arg Max}_{\mathfrak{P}} \sum_{\tau=0}^{\infty} d^\tau E_t \{ R_{t,t+\tau} \pi_{t+\tau}^m (\mathfrak{P} \Pi^\tau) / e_{t+\tau} \}$$

with  $R_{t,t} = 1$  and  $R_{t,t+\tau} = \prod_{k=1}^{\tau-1} (1 + i_{t+k}^*)^{-1}$  for  $\tau > 1$ , where  $i_{t+k}^*$  is the world interest rate between  $t$  and  $t + 1$ . The solution of this decision problem is

$$p_{t,t}^m = (v/(v - 1)) \left\{ \sum_{\tau=0}^{\infty} (d\Pi^{-v})^\tau E_t \Xi_{t,t+\tau}^m P_{t+\tau}^* \right\} / \left\{ \sum_{\tau=0}^{\infty} (d\Pi^{1-v})^\tau E_t \Xi_{t,t+\tau}^m / e_{t+\tau} \right\} \tag{10}$$

with  $\Xi_{t,t+\tau}^m = R_{t,t+\tau} Q_{t+\tau}^m (P_{t+\tau}^m)^v$ .

The price indices  $P_t^d$ ,  $P_t^m$ ,  $P_t^x$  (see (3) and (6)) evolve according to

$$\begin{aligned} (P_t^i)^{1-v} &= d(P_{t-1}^i \Pi)^{1-v} + (1 - d)(p_{t,t}^i)^{1-v}, \quad i = d, m, \\ (P_t^x)^{1-v} &= d(P_{t-1}^x \Pi^*)^{1-v} + (1 - d)(p_{t,t}^x)^{1-v}. \end{aligned} \tag{11}$$

### 2.3. The representative household

Household preferences are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t). \tag{12}$$

$E_t$  denotes the mathematical expectation conditional upon complete information pertaining to period  $t$  and earlier.  $C_t$  and  $L_t$  are period  $t$  consumption and labor effort.  $0 < \beta < 1$  is the subjective discount factor.  $U$  is a utility function given by

$$U(C_t, L_t) = \ln(C_t) - L_t. \tag{13}$$

<sup>6</sup>It might seem preferable to assume that importers discount future payoffs at the intertemporal marginal rate of substitution of foreign households. This would require modeling the consumption behavior of those households—which is beyond the scope of the small open economy model here.

As indicated earlier, the household owns all domestic producers and accumulates physical capital. The law of motion of the capital stock is

$$K_{t+1} + \phi(K_{t+1}, K_t) = K_t(1 - \delta) + I_t, \quad (14)$$

where  $I_t$  is the gross investment,  $0 < \delta < 1$  is the depreciation rate of capital, and  $\phi$  is an adjustment cost function:  $\phi(K_{t+1}, K_t) = \frac{1}{2}\Phi\{K_{t+1} - K_t\}^2/K_t$ ,  $\Phi > 0$ .

The household also holds nominal one-period domestic and foreign currency bonds. Its period  $t$  budget constraint is

$$A_{t+1} + e_t B_{t+1} + P_t(C_t + I_t) = A_t(1 + i_{t-1}) + e_t B_t(1 + i_{t-1}^f) + R_t K_t + \int_0^1 \pi_t^{dx}(s) ds + W_t L_t. \quad (15)$$

$A_t$  and  $B_t$  are net stocks of domestic and foreign currency bonds that mature in period  $t$ , while  $i_{t-1}$  and  $i_{t-1}^f$  are the interest rates on these bonds.

The household chooses a strategy  $\{A_{t+1}, B_{t+1}, K_{t+1}, C_t, L_t\}_{t=0}^{t=\infty}$  to maximize its expected lifetime utility (12), subject to constraints (14) and (15) and to initial values  $A_0, B_0, K_0$ . Ruling out Ponzi schemes, the following equations are first-order conditions of this decision problem:

$$1 = (1 + i_t)E_t\{\rho_{t,t+1}(P_t/P_{t+1})\}, \quad (16)$$

$$1 = (1 + i_t^f)E_t\{\rho_{t,t+1}(P_t/P_{t+1})(e_{t+1}/e_t)\}, \quad (17)$$

$$1 = E_t\{\rho_{t,t+1}(R_{t+1}/P_{t+1} + 1 - \delta - \phi_{2,t+1})/(1 + \phi_{1,t})\}, \quad (18)$$

$$W_t/P_t = C_t, \quad (19)$$

where  $\rho_{t,t+1} = \beta C_t/C_{t+1}$ ,  $\phi_{1,t} = \partial\phi(K_{t+1}, K_t)/\partial K_{t+1}$ ,  $\phi_{2,t+1} = \partial\phi(K_{t+2}, K_{t+1})/\partial K_{t+1}$ . Eqs. (16)–(18) are Euler conditions, and (19) says that the household equates its marginal rate of substitution between consumption and leisure to the real wage rate.

#### 2.4. Uncovered interest parity

Up to a (log-)linear approximation, (16) and (17) imply uncovered interest parity (UIP):  $E_t \ln(e_{t+1}/e_t) \cong i_t - i_t^f$ . Given the well-documented strong and persistent empirical departures from UIP during the post-BW era (e.g., Lewis, 1995), variants of the model are explored in which the Euler condition for foreign currency bonds (17) is disturbed by a stationary exogenous stochastic random variable,  $\varphi_t$  (“UIP shock”, henceforth) whose unconditional mean is unity ( $E\varphi_t = 1$ ):

$$1 = \varphi_t(1 + i_t^f)E_t\{\rho_{t,t+1}(P_t/P_{t+1})(e_{t+1}/e_t)\}. \quad (20)$$

(Up to a (log-)linear approximation, (16) and (20) imply  $E_t \ln(e_{t+1}/e_t) \cong i_t - i_t^f - (\varphi_t - 1)$ .) As discussed in the Appendix,  $\varphi_t$  can be interpreted as reflecting a bias in the household’s date  $t$  forecast of the date  $t+1$  exchange rate,  $e_{t+1}$ . (Frankel and Froot (1989), document biases in exchange rate forecasts; structural models with UIP shocks have, i.e., been studied by Mark and Wu (1998) and Jeanne and Rose



(2000), who interpret these shocks as “fads”, and by McCallum and Nelson (1999, 2000), and Taylor (1993b), who refer to them as “risk premia”.)

2.5. Market clearing conditions

Supply equals demand in intermediate goods markets because intermediate goods firms meet all demand at posted prices. Market clearing for the final good, labor, and rental capital requires:

$$Z_t = C_t + I_t, \quad L_t = \int_0^1 L_t(s) ds, \quad K_t = \int_0^1 K_t(s) ds. \tag{21}$$

$Z_t$ ,  $L_t$  and  $K_t$  are the supplies of the final good, of labor, and of rental capital, respectively, while  $\int_0^1 L_t(s) ds$  and  $\int_0^1 K_t(s) ds$  represent total demand for labor and capital (by intermediate goods producers).

It is assumed that foreigners do not hold bonds denominated in the currency of the small open economy. Thus, market clearing for bonds of this type requires

$$A_t = 0. \tag{22}$$

$i_t^f$ , the interest rate at which the household can borrow/lend foreign currency funds equals the exogenous world interest rate,  $i_t^*$ , plus a “spread” that is a decreasing function of the country’s net foreign asset position:

$$(1 + i_t^f)/\Pi^* = (1 + i_t^*)/\Pi^* - \lambda(B_{t+1}/P_t^*)/\chi, \quad \lambda > 0, \tag{23}$$

where  $\chi$  is the steady-state value of exports, expressed in units of foreign output ( $P_t^* Q_t^*/P_t^*$ ).  $\lambda$  captures the degree of capital mobility—a lower  $\lambda$  represents higher capital mobility. Under perfect mobility ( $\lambda = 0$ ), the country would face an infinite supply of/demand for foreign funds when  $i_t^f \neq i_t^*$ . The simulations assume that financial capital is *not* perfectly mobile (owing to transaction costs or other frictions):  $\lambda > 0$ . This ensures the existence of stationary equilibrium, which allows to solve the model using the Sims (2000) method. (When  $\lambda = 0$ , the model is a version of the permanent income theory of consumption, and net assets and consumption are non-stationary. Schedules similar to (23) have also been assumed by Senhadji (1997) and Schmitt-Grohé and Uribe (2001a).)

2.6. Exogenous variables

Productivity, world inflation, the world interest rate, and the UIP shock follow these processes:

$$\theta_t = (1 - \rho^\theta) + \rho^\theta \theta_{t-1} + \varepsilon_t^\theta, \quad 0 \leq \rho^\theta < 1, \tag{24}$$

$$\Pi_t^* = (1 - \rho^*)\Pi^* + \rho^* \Pi_{t-1}^* + \Pi^* \varepsilon_t^*, \tag{25}$$

where  $\Pi_t^* = P_t^*/P_{t-1}^*$ ,  $0 \leq \rho^* < 1$ ,

$$i_t^* = (1 - \rho^i)i^* + \rho^i i_{t-1}^* + \Pi^* \varepsilon_t^i, \quad 0 \leq \rho^i < 1, \tag{26}$$

$$\varphi_t = (1 - \rho^\varphi) + \rho^\varphi \varphi_{t-1} + \varepsilon_t^\varphi, \quad 0 \leq \rho^\varphi < 1, \tag{27}$$

where  $\varepsilon_t^\theta$ ,  $\varepsilon_t^*$ ,  $\varepsilon_t^i$ , and  $\varepsilon_t^\varphi$  are independent white noises with standard deviations  $\sigma^\theta$ ,  $\sigma^*$ ,  $\sigma^i$ , and  $\sigma^\varphi$ , respectively.

### 2.7. The monetary policy rule

Much recent research on monetary policy regimes has focused on rules that stipulate a response of the interest rate to inflation and to real GDP (e.g., Taylor, 1993a, 1999). The baseline rule considered here is

$$i_t = i + \Gamma_\pi \widehat{\Pi}_t^d + \Gamma_y \widehat{Y}_t, \quad (28)$$

with  $\widehat{\Pi}_t^d = (\Pi_t^d - \Pi)/\Pi$ ,  $\widehat{Y}_t = (Y_t - Y)/Y$ , where  $\Pi_t^d = P_t^d/P_{t-1}^d$  is the growth factor of the price index of domestic intermediate goods that are sold in the domestic market—(gross) domestic PPI inflation.  $Y_t$  is real GDP (measurement of GDP: see Appendix).  $i$  and  $Y$  are the steady-state nominal interest rate and steady-state GDP, respectively. Throughout the paper, variables without time subscripts denote steady-state values, and  $\hat{x}_t = (x_t - x)/x$  is the relative deviation of a variable  $x_t$  from its steady-state value,  $x$ .  $\Gamma_\pi$  and  $\Gamma_y$  in (28) are parameters.

The central bank irrevocably commits to setting  $\Gamma_\pi$  and  $\Gamma_y$  at the values that maximize the unconditional expected value of household utility,  $E(U(C_t, L_t))$ . Note that a fully optimal feedback rule would stipulate a response of the interest rate to all current and lagged state variables (e.g., Clarida et al., 1999; Rotemberg and Woodford, 1997). I focus on a “simple” rule such as (28) because: (i) simple rules appear to capture quite well actual central bank behavior (e.g., Taylor, 1993a, 1999); (ii) the use of a simple rule facilitates commitment as the public can easily monitor whether the central bank sticks to such a rule; (iii) computationally, it does not seem feasible to determine the fully optimal rule for the complex model considered here.<sup>7</sup>

### 2.8. Solution method, welfare measures

The model is solved using Sims’ (2000) second-order accurate method (see Appendix), and  $E(U(C_t, L_t))$  is maximized numerically with respect to the policy parameters  $\Gamma_\pi$  and  $\Gamma_y$  (attention is restricted to parameter values for which a unique stationary equilibrium exists).

A second-order Taylor expansion of the utility function around the steady state gives  $E(U(C_t, L_t)) \cong U(C, L) + E(\widehat{C}_t) - LE(\widehat{L}_t) - Var(\widehat{C}_t)$ , where  $Var(\widehat{C}_t)$  is the variance of  $\widehat{C}_t$ . (For the parameter values used below,  $L = 0.74$ .)

In what follows, welfare is expressed as the permanent relative change in consumption (compared to the steady state),  $\zeta$ , that yields expected utility  $E(U(C_t, L_t))$ :  $U((1 + \zeta)C, L) = U(C, L) + E(\widehat{C}_t) - LE(\widehat{L}_t) - Var(\widehat{C}_t)$ .  $\zeta$  can be

<sup>7</sup>The optimal rule can, in principle, be found by selecting the path of the interest rate (and of the other endogenous variables) that maximizes welfare subject to the equilibrium conditions of the economy (e.g., Clarida et al., 1999). For the model here, this “Ramsey problem” is not a concave programming problem and hence is intractable. Solving the system of equations obtained by setting to zero the derivatives of the Lagrangian associated with the Ramsey problem is not feasible using the Sims algorithm.

decomposed into components, denoted by  $\zeta^m$  and  $\zeta^v$ , that reflect the means of consumption and hours worked, and the variance of consumption, respectively:

$$U((1 + \zeta^m)C, L) = U(C, L) + E(\widehat{C}_t) - LE(\widehat{L}_t),$$

$$U((1 + \zeta^v)C, L) = U(C, L) - Var(\widehat{C}_t).$$

Eq. (13) implies  $\ln(1 + \zeta) = E(\widehat{C}_t) - LE(\widehat{L}_t) - Var(\widehat{C}_t)$ ,  $\ln(1 + \zeta^m) = E(\widehat{C}_t) - LE(\widehat{L}_t)$ ,  $\ln(1 + \zeta^v) = -Var(\widehat{C}_t)$  and thus  $(1 + \zeta) = (1 + \zeta^m)(1 + \zeta^v)$ .

## 2.9. Parameters (non-policy)

Following Kollmann (2001a), the model is calibrated to quarterly data for Japan, Germany and the U.K. The steady-state domestic and foreign *real* interest rates are assumed to be identical,  $r \equiv (1 + i)/\Pi - 1 = (1 + i^*)/\Pi^* - 1$ , which implies that the steady-state net asset position is zero.  $r$  is set at  $r = 0.01$ , a value that corresponds roughly to the long-run average return on capital. The subjective discount factor is, hence, set at  $1/(1.01)$ , since  $\beta(1 + r) = 1$  holds in steady state.

The price elasticities of the country's aggregate imports and exports (see (2) and (5)) are set at  $\varphi = \eta = 0.6$ . This is the median value of the estimates of  $\varphi$  and  $\eta$  for the three sample countries reported by Hooper and Marquez (1995).  $\alpha^m$  (see (1)) is set so that the steady-state imports/GDP ratio is 30%, consistent with German and U.K. data (for Japan, the imports/GDP ratio  $\approx 10\%$ ; the key results continue to hold when a 10% ratio is assumed).

The steady-state price-marginal cost markup factor for intermediate goods is set at  $v/(v - 1) = 1.2$ , consistent with the findings of Martins et al. (1996) for the three sample countries. The technology parameter  $\psi$  (see (4)) is set at  $\psi = 0.24$ , which entails a 60% steady-state labor income/GDP ratio, consistent with data for these countries. Aggregate data suggest a quarterly capital depreciation rate of about 2.5%; thus,  $\delta = 0.025$  is used. The capital adjustment cost parameter  $\Phi$  is set at  $\Phi = 15$  in order to match the fact that the standard deviation of HP filtered log investment is 3–4 times larger than that of GDP in the sample countries.

Panel regressions (for 21 OECD countries) of cross-country interest rate differentials on net foreign asset positions reported by Lane and Milesi-Ferretti (2001) suggest a value  $\lambda = 0.0019$  for the capital mobility parameter  $\lambda$  in (23). (See the discussion in the Appendix.)

Lopez-Salido (2000) fits a Calvo-style price setting equation to German and U.K. data. His estimates suggest that the average price-change-interval is about 4 quarters. Hence,  $d$  is set at  $d = 0.75$ . The steady-state growth factors of the domestic and world price levels are set at  $\Pi = \Pi^* = 1$ .  $\Pi$  and  $\Pi^*$  have no effect on real variables, because of indexing, and because the innovations to  $i_t^*$  and  $\Pi_t^*$  are scaled by  $\Pi^*$  in (25) and (26).

Fitting (24) to (geometrically detrended) quarterly total factor productivity (TFP) for the three sample countries (1973–1994) yields (average) parameter estimates of:  $\rho^\theta = 0.9$ ,  $\sigma^\theta = 0.01$ . Using the U.S. CPI and the U.S. 3-month CD rate (1973–1994) as measures of the world price level  $P_t^*$  and interest rate  $i_t^*$ , gives the following

estimates of the parameters of (25) and (26):  $\rho^* = 0.8$ ,  $\sigma^* = 0.005$ ,  $\rho^i = 0.75$ ,  $\sigma^i = 0.004$ . Kollmann (2002) constructs estimates of quarterly deviations from UIP between the U.S. and each of the three sample countries; fitting (27) to these estimated  $\varphi_t$  series yields (average) parameter estimates of:  $\rho^\varphi = 0.5$ ,  $\sigma^\varphi = 0.033$ . The simulations use the parameter estimates for Eqs. (24)–(27) that were just reported.<sup>8</sup>

### 3. Results

Tables 1–2 show results for the baseline sticky-prices model, as well as for a structure with flexible prices (in which the price adjustment parameter is set at  $d = 0$ ). Table 3 considers a variant of the sticky-prices model with a pegged exchange rate. Table 4 compares historical and predicted statistics.

In the tables,  $\Pi_t = P_t/P_{t-1}$  is gross CPI (final good) inflation.  $\Delta e_t = e_t/e_{t-1}$  is the depreciation factor of the nominal exchange rate.  $\mu_t^d = P_t^d/MC_t$ ,  $\mu_t^x = e_t P_t^x/MC_t$ , and  $\mu_t^m = P_t^m/(e_t P_t^*)$  are geometric averages of the markup factors of individual domestic intermediate goods producers in the domestic market and in the export market, and of the markup factors of individual importers, respectively (e.g.,  $\mu_t^d = \left\{ \int_0^1 \mu_t^d(s)^{1-\nu} ds \right\}^{1/(1-\nu)}$ , where  $\mu_t^d(s) = p_t^d(s)/MC_t$ ).  $RER_t = e_t P_t^*/P_t$  is the real exchange rate.  $NFA_t$  is the net foreign asset position, expressed in units of the aggregate import good and normalized by steady-state GDP (in units of the import good). ( $NFA_t = (B_{t+1}/P_t^*)/Y$ , where  $Y$  is the steady-state value of  $Y_t^{\text{nom}}/(e_t P_t^*)$ , with  $Y_t^{\text{nom}}$ : nominal GDP; see Appendix).

Predicted standard deviations and mean values of these (and other) variables are shown, as well as impulse responses (Table 2). The variables are quarterly.<sup>9</sup> In Tables 1–3, the statistics/responses for the domestic interest rate ( $i_t$ ) and  $NFA_t$  refer to differences of these variables from steady-state values ( $i_t$  is a quarterly rate expressed in fractional units), while statistics/responses for the remaining variables refer to relative deviations from steady-state values. All statistics/responses are expressed in percentage terms.<sup>10</sup> Results are presented for simulations in which the economy is subjected to just one type of shock (see Cols. labeled “Shocks to  $\theta$ ”, “Shocks to  $\tilde{r}^*$ ”, “Shocks to  $\varphi$ ”, and “Shocks to  $P^*$ ”) and for simulations in which the economy is simultaneously subjected to the four shocks (Cols. labeled “Shocks to  $\theta, \tilde{r}^*, \varphi, P^*$ ”).

#### 3.1. Results for the baseline sticky-prices model (Table 1, cols. 1–5)

##### 3.1.1. Combined effect of shocks (Table 1, Col. 1)

The optimized policy rule, with the four simultaneous shocks, has inflation and output coefficients of 3.01 and  $-0.01$ , respectively. The rule thus prescribes a strong

<sup>8</sup>Taylor (1993b) reports estimated standard deviations of UIP innovations of the U.S. vis-à-vis the other G7 countries that range between 0.037 and 0.101.

<sup>9</sup>The net foreign assets measure used in Table 4 is annual (i.e. sampled every fourth period).

<sup>10</sup>For example, the standard deviation and the mean of  $\hat{C}_t = (C_t - C)/C$  are  $0.0204 = 2.04\%$  and  $0.0035 = 0.35\%$ , respectively, in the baseline sticky-prices model (see Col. 1, Table 1).

Table 1

Baseline model. Optimized policy rule:  $i_t = i + 3.01\hat{\Pi}_t^d - 0.01 \hat{Y}_t$

	Sticky prices					Flexible prices				
	Shocks to					Shocks to				
	$\theta, i^*, \varphi, P^*$	$\theta$	$i^*$	$\varphi$	$P^*$	$\theta, i^*, \varphi, P^*$	$\theta$	$i^*$	$\varphi$	$P^*$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<b>Standard deviations (in %)</b>										
<i>Y</i>	2.29	2.26	0.11	0.26	0.25	2.57	2.44	0.21	0.71	0.28
<i>C</i>	2.04	1.80	0.31	0.78	0.48	2.87	1.39	0.65	2.26	0.87
<i>I</i>	4.18	3.81	0.59	1.33	0.89	6.92	2.98	1.56	5.69	2.03
$\Pi$	0.23	0.08	0.06	0.18	0.08	2.24	0.37	0.43	2.10	0.51
$\Pi^d$	0.08	0.08	0.00	0.00	0.00	0.09	0.07	0.01	0.04	0.01
<i>i</i>	0.29	0.28	0.01	0.01	0.01	0.31	0.29	0.01	0.10	0.02
$\Delta e$	7.56	1.19	1.43	7.03	2.06	7.53	1.26	1.43	6.98	2.10
<i>RER</i>	7.63	2.62	1.76	6.56	2.29	5.86	2.51	1.36	4.79	1.81
<i>NFA</i>	16.38	3.34	5.15	12.70	8.33	13.52	4.51	4.33	10.06	6.51
$\mu^d$	0.11	0.11	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00
$\mu^x$	6.62	1.42	1.41	5.87	2.32	0.00	0.00	0.00	0.00	0.00
$\mu^m$	6.41	1.33	1.40	5.86	1.73	0.00	0.00	0.00	0.00	0.00
<b>Means (in %)</b>										
<i>Y</i>	0.01	-0.00	-0.01	0.02	-0.00	-0.05	0.00	-0.01	-0.03	-0.02
<i>C</i>	0.35	-0.01	0.03	0.21	0.12	0.39	-0.01	0.03	0.27	0.09
$Q^d$	0.14	0.00	0.01	0.08	0.04	0.15	0.00	0.01	0.10	0.03
$Q^m$	0.86	-0.00	0.08	0.50	0.28	1.10	0.01	0.08	0.77	0.23
$Q^x$	-0.74	-0.06	-0.06	-0.27	-0.35	-0.54	0.00	-0.05	-0.34	-0.15
<i>L</i>	-0.07	0.00	-0.01	-0.03	-0.03	-0.15	-0.00	-0.01	-0.09	-0.04
<i>K</i>	0.29	-0.00	0.02	0.18	0.09	0.25	-0.00	0.02	0.18	0.05
<i>RER</i>	-0.32	-0.02	-0.04	-0.08	-0.17	-0.54	-0.03	-0.06	-0.45	-0.15
<i>NFA</i>	21.79	0.03	1.13	17.17	3.46	23.50	0.05	1.21	18.43	3.79
$\mu^d$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\mu^x$	0.38	0.06	0.01	0.10	0.21	0.00	0.00	0.00	0.00	0.00
$\mu^m$	0.16	0.05	0.01	0.09	0.01	0.00	0.00	0.00	0.00	0.00
<b>Welfare (% equivalent variation in consumption)</b>										
$\zeta$	0.39	-0.02	0.04	0.23	0.14	0.46	-0.02	0.04	0.31	0.12
$\zeta^m$	0.41	-0.01	0.04	0.23	0.14	0.50	-0.01	0.04	0.34	0.12
$\zeta^v$	-0.02	-0.01	-0.00	-0.00	-0.00	-0.04	-0.01	-0.00	-0.03	-0.00

Notes:  $\theta$ : productivity;  $i^*$ : world interest rate;  $\varphi$ : UIP shock;  $P^*$ : world price level; *Y*: GDP; *C*: consumption; *I*: investment;  $\Pi$ : gross CPI (final good) inflation;  $\Pi^d$ : gross domestic PPI inflation; *i*: domestic nominal interest rate;  $\Delta e$ , depreciation factor of nominal exchange rate; *RER*: real exchange rate; *NFA*: net foreign assets (expressed in units of foreign good and normalized by steady state GDP);  $\mu^d$ ,  $\mu^x$ , and  $\mu^m$ : average markup factors of domestic intermediate goods producers in the domestic market and in the export markets, and of importers;  $Q^d$  and  $Q^x$ : domestic intermediate goods sold domestically and exported;  $Q^m$ : imports; *L*: hours worked; *K*: capital stock;  $\zeta$ ,  $\zeta^m$ , and  $\zeta^v$ : welfare measures.

Standard deviations and means of *i* and *NFA* refer to differences from steady-state values; statistics for the remaining variables refer to relative deviations from steady-state values. All statistics have been multiplied by 100, i.e. expressed in percentage terms.

rise in the interest rate in response to an increase in domestic PPI inflation; in contrast, the response coefficient on output is close to zero. The implied standard deviation of domestic PPI inflation is low: 0.08%; the predicted standard deviation of CPI inflation is somewhat higher: 0.23%. Hence, the optimized rule has a rather strict anti-inflation stance.

With the four simultaneous shocks (and the optimized rule), the predicted standard deviations of output and consumption are about 2%. The predicted standard deviations of nominal and real exchange rates (about 7.5%) are much higher. The net foreign asset position (normalized by steady-state GDP) has the highest standard deviation among the variables considered in the table: 16.38%.

Mean GDP and mean hours worked differ only very slightly from steady-state values. Mean consumption and the mean capital stock are about 0.3% above steady state, and the mean stock of foreign assets exceeds the steady-state stock by an amount that corresponds to 21.79% of steady-state GDP. Mean imports are 0.86% above steady-state (74% of the increase in (mean) consumption is met out of increased imports of intermediate goods). The mean real exchange rate exhibits a 0.32% appreciation relative to steady state.

Welfare is higher in the stochastic economy (under the optimized policy rule) than in steady state:  $\zeta = 0.39\%$ . The welfare gain is mainly driven by the increase in mean consumption,  $\zeta^m = 0.41\%$ . In contrast, the welfare cost of consumption variance is negligible,  $\zeta^v = -0.02\%$ .

### 3.1.2. The sticky-prices economy subjected to each type of shock separately

In order to interpret the preceding results, it is useful to consider the behavior of the sticky-prices economy when it is subjected to each of the four types of shocks separately—see Cols. 2–5, Table 1. (Cols. 2–5 assume the same policy parameters as Col. 1:  $\Gamma_\pi = 3.01$ ,  $\Gamma_y = -0.01$ ). These simulations show that productivity shocks account for 98% of the variance of GDP (under four simultaneous shocks), for about 80% of the variance of consumption and investment, and for 99% of the variance of the domestic interest rate. UIP shocks explain about 80% of the variance of nominal and real exchange rates, and 60% of the variance of net foreign assets.

UIP shocks also explain most of the increase in mean foreign asset holdings, mean consumption and welfare (welfare with only UIP shocks:  $\zeta = 0.23\%$ ). Shocks to world inflation too have a noticeable positive effect on mean foreign assets and welfare, while the effect of productivity and world interest rate shocks on mean values and on welfare is negligible.

### 3.1.3. Explaining the mean foreign asset position

Eq. (20) implies  $1 = E\{\varphi_t(1 + i_t^f)q_{t+1}\}$ , where  $q_{t+1} = \beta(RER_{t+1}/RER_t)(C_t/C_{t+1})/\Pi_{t+1}^*$  is the household's marginal rate of substitution between units of the foreign currency available at  $t$  and  $t+1$ . As shown in the Appendix, an increase in the variance of  $q_{t+1}$  raises the (unconditional) expected value of  $q_{t+1}$ , which leads the country to hold more foreign currency bonds—which implies that it imports and consumes more (on average). UIP shocks are the main source of real exchange rate movements. This explains why these shocks have the strongest (positive) effect on

mean net foreign assets and consumption—and on welfare (among the four types of shocks).

### 3.2. Results for the flex-prices variant of the model (Table 1, cols. 6–10)

In the flex-prices variant, monetary policy does not affect real variables (Cols. 6–10 assume the same policy parameters as Cols. 1–5:  $\Gamma_\pi = 3.01$ ,  $\Gamma_y = -0.01$ ). The means and variances of real variables are, mostly, rather similar across the sticky- and flex-prices variants of the model (however, markups are constant under flexible prices). Note that in the flex-prices variant too, UIP shocks induce significant nominal and real exchange rate volatility (UIP shocks again are the dominant source of fluctuations in nominal and real exchange rates) and markedly increase the country's demand for foreign currency bonds. Welfare, with the four simultaneous shocks, is only slightly higher under flexible prices,  $\zeta = 0.46\%$ , than under sticky prices,  $\zeta = 0.39\%$ .

#### 3.2.1. Impulse responses (Table 2)

Table 2 shows that dynamic responses to shocks are broadly similar (at least qualitatively) across the sticky-prices structure (with optimized policy) and the flex-prices structure. In both structures, a positive productivity shock lowers the domestic interest rate and raises GDP; consumption rises in response to positive shocks to productivity and to world inflation, and falls in response to positive UIP and world interest rate shocks; positive productivity, world interest rate and UIP shocks induce, on impact, a depreciation of nominal and real exchange rates, while a positive world inflation shock induces an appreciation.

The finding that the optimized monetary policy rule prescribes a procyclical response to productivity shocks is consistent with the previous literature (e.g., Ireland, 1996): economic efficiency requires an immediate increase in output when the economy receives a positive productivity shock; price stickiness dampens the (immediate) expansion of output; procyclical monetary policy helps to overcome that sluggishness of the output response.

### 3.3. Why is welfare so similar under sticky prices (with optimized rule) and under flexible prices?

A policy that perfectly stabilizes domestic PPI inflation in the sticky-prices structure (by setting  $\Gamma_\pi = \infty$ ) yields only minimally lower welfare than the optimized rule (28):  $\zeta = 0.387\%$  vs.  $\zeta = 0.389\%$  (optimized rule). Perfect stabilization of domestic PPI inflation entails that the markup factor of domestic intermediate goods producers in the home market is constant and equal to its steady-state value:  $\mu_t^d = P_t^d/MC_t = v/(v-1)$ .<sup>11</sup> (Analogously, full stabilization of

<sup>11</sup> Under full domestic PPI inflation stabilization,  $p_{t,t}^d = P_t^d = \Pi^d P_0^d$  holds, and (8) implies  $\Pi^d P_0^d = F_t^d/G_t^d$ , with  $F_t^d = (\Pi^d P_0^d)^v Q_t^d (v/(v-1)) MC_t/P_t + d\Pi^{1-v} E_t \rho_{t,t+1} F_{t+1}^d$ ,  $G_t^d = (\Pi^d P_0^d)^v Q_t^d/P_t + d\Pi^{1-v} E_t \rho_{t,t+1} G_{t+1}^d$ . This implies  $\mu_t^d = \Pi^d P_0^d/MC_t = P_t^d/MC_t = v/(v-1)$ .

Table 2

Baseline model: % responses to 1 standard deviation innovations

	<i>Y</i>	<i>C</i>	<i>I</i>	<i>L</i>	<i>P</i>	<i>P</i> <sup>d</sup>	<i>NFA</i>	<i>e</i>	<i>RER</i>	<i>i</i>	Exogenous variables
<b>(a) Sticky prices</b>											
<i>(i) Productivity shock</i>											
$\tau = 0$	0.86	0.92	2.08	-0.17	0.04	-0.04	-0.02	1.15	1.11	-0.11	$\theta$ 1.00
$\tau = 4$	0.63	0.48	0.99	-0.07	0.05	-0.15	0.19	0.72	0.67	-0.08	0.65
$\tau = 24$	0.13	0.04	-0.02	0.01	-0.23	-0.33	-0.52	-0.08	0.17	-0.01	0.08
<i>(ii) World interest rate shock</i>											
$\tau = 0$	0.01	-0.05	-0.16	0.02	0.05	-0.00	0.38	1.29	1.25	-0.00	$i^*$ 0.40
$\tau = 4$	0.01	-0.06	-0.18	0.02	0.05	0.00	0.99	0.23	0.18	0.00	0.12
$\tau = 24$	-0.02	0.05	0.06	-0.02	-0.03	0.01	0.69	-0.13	-0.10	0.00	0.00
<i>(iii) UIP shock</i>											
$\tau = 0$	0.05	-0.17	-0.52	0.06	0.15	0.00	1.66	5.87	5.72	0.00	$\varphi$ 3.30
$\tau = 4$	0.01	-0.04	-0.22	0.03	0.04	0.01	2.74	-0.16	-0.20	0.00	0.20
$\tau = 24$	-0.04	0.10	0.13	-0.05	-0.07	0.02	1.44	-0.26	-0.20	0.00	0.00
<i>(iv) Shock to world inflation</i>											
$\tau = 0$	0.02	0.04	0.22	0.03	-0.06	0.00	-0.46	-2.06	-1.50	0.00	$P^*$ 0.50
$\tau = 4$	0.08	0.06	0.27	0.09	-0.07	0.01	-1.40	-2.07	-0.31	0.00	1.68
$\tau = 24$	0.04	-0.08	-0.10	0.05	0.09	0.22	-1.19	-2.23	0.16	0.00	2.49
<b>(b) Flexible prices</b>											
<i>(i) Productivity shock</i>											
$\tau = 0$	1.00	0.63	1.45	0.01	0.33	-0.04	-0.11	1.22	0.88	-0.13	$\theta$ 1.00
$\tau = 4$	0.68	0.40	0.97	0.01	0.13	-0.14	-0.45	0.79	0.65	-0.08	0.65
$\tau = 24$	0.12	0.03	-0.01	0.01	-0.25	-0.33	-0.68	-0.05	0.20	-0.01	0.08
<i>(ii) World interest rate shock</i>											
$\tau = 0$	0.13	-0.44	-1.10	0.17	0.39	-0.00	0.26	1.30	0.90	0.01	$i^*$ 0.40
$\tau = 4$	0.01	-0.10	-0.25	0.03	0.09	0.01	0.77	0.26	0.17	0.00	0.12
$\tau = 24$	-0.02	0.04	0.07	-0.02	-0.02	0.02	0.60	-0.10	-0.09	0.00	0.00
<i>(iii) UIP shock</i>											
$\tau = 0$	0.61	-1.98	-5.09	0.80	1.77	0.01	1.15	5.82	4.04	0.11	$\varphi$ 3.30
$\tau = 4$	-0.05	-0.02	-0.03	-0.00	0.04	0.07	2.07	-0.03	-0.07	0.01	0.20
$\tau = 24$	-0.04	0.07	0.14	-0.04	-0.00	0.08	1.19	-0.17	-0.18	0.07	0.00
<i>(iv) Shock to world inflation</i>											
$\tau = 0$	-0.16	0.54	1.32	-0.21	-0.47	0.00	-0.32	-2.08	-1.11	-0.01	$P^*$ 0.50
$\tau = 4$	-0.03	0.17	0.41	-0.06	-0.14	-0.01	-1.03	-2.13	-0.31	-0.01	1.68
$\tau = 24$	0.03	-0.05	-0.10	0.03	0.03	-0.02	-0.94	-2.32	0.13	0.00	2.49

Notes:  $\tau$ : periods after shock. Columns labeled *Y*, *C*, etc. show responses of the corresponding variables. *P*: price of final good; *P*<sup>d</sup>: domestic PPI; *e*: nominal exchange rate; the remaining variables are defined in Table 1.

The impulse responses are generated as follows. At a given date, say *T*, all state variables are set at steady state values. A “baseline” path for the endogenous variables is computed (using (A.2), (A.3) in the Appendix) by setting all exogenous innovations to zero in periods  $t \geq T$ . Then responses to one-time 1 standard deviation exogenous innovation at *T* are computed; the table reports differences/relative deviations (that have been multiplied by 100, i.e. expressed in percentage terms) of these responses from the “baseline” path (responses of interest rates (*i*, *i*<sup>\*</sup>) and net foreign assets position (*NFA*): differences from baseline path; responses of remaining variables: relative deviations from baseline path).



$P_t^m [P_t^x]$  inflation would entail  $\mu_t^m = v/(v-1)[\mu_t^x = v/(v-1)]$ . As the optimized rule implies near (but not perfect) domestic PPI inflation stabilization, the standard deviation of  $\hat{\mu}_t^d = (\mu_t^d - \mu)/\mu$  is low (0.11%), and its expected value is (basically) zero. In contrast, the markup factors for exports and imports have high standard deviations (about 6%) under the optimized rule, which mainly reflects the exchange rate movements induced by UIP shocks.

Under price flexibility, by contrast, markup factors always equal their steady-state value:  $\mu_t^d = \mu_t^m = \mu_t^x = v/(v-1)$ . If monetary policy in the sticky-prices economy could simultaneously set the growth rates of  $P_t^d, P_t^m$  and  $P_t^x$  at the corresponding steady-state values, then that policy would replicate the flex-prices equilibrium. It appears that simultaneous stabilization of these three inflation rates (and replication of the flex-prices equilibrium) is impossible in the baseline sticky-prices structure (with pricing-to-market).<sup>12</sup> However, domestic intermediate goods production accounts for 70% of domestic absorption (on average). This helps to understand why the near stabilization of domestic PPI inflation (under sticky prices) yields welfare results that are rather similar to those in the flex-prices economy.

### 3.4. Exchange rate peg

As the optimized rule implies significant nominal and real exchange rate volatility, one might suspect that pegging the exchange rate noticeably lowers welfare. This is confirmed by Cols. 1–2 in Table 3, where a variant of the sticky-prices model with a peg is considered. With the four simultaneous shocks, welfare is  $\zeta = -0.38\%$  under the peg, compared to  $\zeta = 0.39\%$  under the optimized rule. Interestingly, both the “means component” of the welfare measure ( $\zeta^m = -0.16\%$ ) and the “variance component” ( $\zeta^v = -0.22\%$ ) are lower under the peg.

The peg greatly raises the variability of consumption and output. Under the peg, the gross domestic interest rate responds (virtually) one-to-one to UIP shocks and to world interest rate shocks—that response is much stronger than under the optimized rule. (Eqs. (16), (20) and (23) imply that  $(1+i_t) = (1+i_t^* - \Pi^* \lambda(B_{t+1}/P_t^*)/\chi)\phi_t$ , under the peg.) Thus, these shocks (especially the UIP shocks, because of the high standard deviation of the latter) have a much more destabilizing effect under the peg.

Mean foreign assets (and mean imports) under the peg (with the four simultaneous shocks) are about as high as under the optimized policy rule—in both regimes, this mainly reflects the influence of UIP shocks.<sup>13</sup> However, mean consumption is noticeably lower ( $E\hat{C}_t = -0.17\%$  under peg;  $E\hat{C}_t = 0.35\%$  under optimized float).

<sup>12</sup>If domestic PPI ( $P_t^d$ ) inflation is stabilized, then marginal cost grows at the constant rate  $\Pi - 1$ . Stabilization of export price inflation then requires a fixed nominal exchange rate (see (9)). Yet, stabilization of import price ( $P_t^m$ ) inflation (and thus of  $\mu_t^m = P_t^m/(e_t P_t^*)$ ) requires a flexible exchange rate.

<sup>13</sup>Under the optimized rule, UIP shocks raise the country's demand for foreign currency bonds because the high real exchange rate volatility caused by these shocks increases the household's expected intertemporal marginal rate of substitution in foreign currency,  $E q_{t+1}$  (see Section 3.1). Under the peg, the real exchange rate is markedly less volatile; however, consumption is much more volatile, when there are UIP shocks—and  $E q_{t+1}$  is roughly as high as under the optimized rule (which explains why the country's demand for foreign currency bonds remains sizable).

This is driven by a reduction in the (mean) output of domestic intermediate goods, compared to the float ( $E\hat{Q}_t^d = -0.47\%$  under peg,  $E\hat{Q}_t^d = 0.14\%$  under float). The latter appears to be due to the fact that the expected markup factor of domestic intermediate goods firms in the domestic market is higher under the peg ( $E\hat{\mu}_t^d = 1.21\%$  under peg;  $E\hat{\mu}_t^d = 0.00\%$  under float).

Under price flexibility ( $d = 0$ ), intermediate goods prices equal current marginal cost multiplied by the constant markup factor  $v/(v - 1) > 1$ . When prices are sticky ( $d > 0$ ), prices set at a given date depend also on (the distribution of) future marginal costs: these prices are increasing functions of the covariance between future marginal cost and future demand (see the Appendix). Under the peg, UIP shocks induce strong responses in domestic demand for intermediate goods and in domestic marginal cost; furthermore, these responses are highly positively correlated: domestic intermediate goods firms typically face high [low] marginal costs in states of the world in which domestic demand for their good (and hence their output) is high [low].<sup>14</sup> These firms therefore set higher markups (in the domestic market).

#### 3.4.1. Effects of a peg that eliminates UIP shocks

A key question in modeling an exchange rate peg is whether it affects the variance of the UIP shocks. Departures from interest rate parity were markedly smaller in the BW era than in the post-BW period (e.g., Kollmann, 2002). This finding is not surprising—under a (credible) peg there is much less scope for irrational exchange rate forecasts than under a float.

Col. 2 in Table 3 (where the variance of the UIP shocks is set to zero) shows that pegging the exchange rate continues to reduce welfare (compared to the optimized rule) when the peg eliminates the UIP shocks: under a peg without UIP shocks, the standard deviation of consumption (2.92%) is higher, and mean consumption ( $E\hat{C}_t = -0.15\%$ ) is lower than under the optimized float (with UIP shocks). (The lower mean consumption under the peg (without UIP shocks) is, i.a., due to the fact that the mean asset position is markedly lower when the variance of the UIP shocks is zero.)

### 3.5. Other variants of the sticky-prices model

#### 3.5.1. Alternative assumptions about price adjustment

Motivated by the empirical failure of the LOP, the baseline model has assumed pricing-to-market (PTM) behavior by firms. In contrast, previous research on optimal monetary policy in open economies has often assumed producer currency

<sup>14</sup> Under the peg, a 1 standard deviation (3.3%) UIP shock lowers aggregate output by 4.3% on impact; the aggregate demand for domestic intermediate goods ( $Q_t^d$ ) and marginal cost ( $MC_t$ ) fall by 6.5% and 7.2%, respectively. (Under the optimized policy rule,  $Q_t^d$  and  $MC_t$  fall by merely 0.1% and 0.01%, in response to the same shock.)

Table 3  
Other variants of the sticky-prices model

	Exchange rate peg		Producer currency pricing
	Shocks to		Shocks to
	$\theta, i^*, \varphi, P^*$	$\theta, i^*, P^*$	$\theta, i^*, \varphi, P^*$
	(1)	(2)	(3)
<b>Standard deviations (in %)</b>			
<i>Y</i>	5.52	2.78	2.63
<i>C</i>	6.66	2.92	2.72
<i>I</i>	16.95	7.42	6.50
$\Pi$	0.96	0.81	2.32
$\Pi^d$	1.17	0.91	0.07
<i>i</i>	3.86	0.59	0.21
$\Delta e$	0.00	0.00	7.79
<i>RER</i>	3.01	2.36	6.01
<i>NFA</i>	18.00	11.23	13.23
$\mu^d$	7.99	3.52	0.33
$\mu^x$	7.99	3.52	0.33
$\mu^m$	0.97	0.97	0.00
<b>Means (in %)</b>			
<i>Y</i>	-0.09	-0.08	-0.06
<i>C</i>	-0.17	-0.15	0.38
$Q^d$	-0.47	-0.29	0.14
$Q^m$	0.85	0.23	1.12
$Q^x$	-0.87	-0.58	-0.56
<i>L</i>	-0.01	-0.02	-0.16
<i>K</i>	-0.10	-0.16	0.23
<i>RER</i>	-1.39	-0.72	-0.74
<i>NFA</i>	20.85	4.38	23.65
$\mu^d$	1.21	0.42	0.01
$\mu^x$	0.56	0.31	0.01
$\mu^m$	0.24	0.24	0.00
<b>Welfare (% equivalent variation in consumption)</b>			
$\zeta$	-0.38	-0.18	0.47
$\zeta^m$	-0.16	-0.14	0.51
$\zeta^v$	-0.22	-0.04	-0.04

Notes: see Table 1.

pricing (PCP); PCP implies that the LOP holds and that exchange rate movements are fully passed through into import prices (expressed in buyer currency).

Aoki (2001), Devereux and Engel (2000), and Galí and Monacelli (2000), i.a., have shown that when price stickiness in producer currency is the only economic distortion (which implies that the flex-prices equilibrium of the economy is efficient), then welfare maximizing monetary policy entails perfect stabilization of domestic PPI inflation (that policy replicates the flex-prices equilibrium).

Col. 3 in Table 3 considers a variant of the model with PCP.<sup>15</sup> There, the optimized rule has response coefficients on domestic PPI inflation and output of  $\Gamma_\pi = 1.52$  and  $\Gamma_y = -0.06$ , respectively. Most predictions are quite close to those generated by the flex-prices structure; note, especially, that the implied standard deviation of domestic PPI inflation is again very low (0.07%).

Under PCP, the central bank can replicate the flex-prices equilibrium by perfectly stabilizing domestic PPI inflation.<sup>16</sup> The flex-prices equilibrium in this economy is *not* efficient (from the household's perspective), because of monopolistic price setting in the domestic intermediate goods markets;<sup>17</sup> also, the country does not fully exploit its market power in the export market, and in the foreign currency bond market.<sup>18</sup> In fact, the optimized monetary policy rule under PCP does not fully stabilize domestic PPI inflation, and it achieves welfare ( $\zeta = 0.47\%$ ) that is slightly higher than welfare under flexible prices ( $\zeta = 0.46\%$ ).

The good welfare properties of (near) domestic PPI inflation stabilization are thus robust to the departures from efficiency considered here. Furthermore, that policy is also desirable under the more realistic PTM assumption (as the discussion of the baseline model has shown).

### 3.5.2. Alternative policy rules

Experiments with other interest rate rules yielded, at best, modest welfare gains, compared to domestic PPI inflation targeting based on the “simple” rule (28). Because of space constraints, only rules that permit a direct response of  $i_t$  to the exchange rate are discussed here, as well as rules that target CPI inflation or the growth rate of the GDP deflator (“GDP inflation”).

Adding a term  $\Gamma_e(e_t - e_{t-1})/e_{t-1}$  to the right-hand side of (28) yields an optimized response coefficient  $\Gamma_e = -0.01$  that is close to zero; the welfare gain is minuscule ( $\zeta$  increases by 0.002%), and the standard deviation of the nominal exchange rate ( $\Delta e_t$ ) is hardly affected: 7.66%. (Including the *level* of the nominal exchange rate and of the domestic PPI in the rule yields roughly similar results.)

Interest in rules that target CPI or GDP inflation is motivated by the fact that the mandate of the ECB is *CPI* stability, while Taylor's (1993a) widely discussed interest

<sup>15</sup> Under PCP,  $p_t^d(s) = e_t p_t^x(s)$ ,  $P_t^d = e_t P_t^x$  holds and the profit of a domestic intermediate good producer is  $\pi_t^{\text{dx}}(p_t^d(s)) = (p_t^d(s) - MC_t)(p_t^d(s)/P_t^d)^{-\nu} [Q_t^d + Q_t^x]$ ; (8) is replaced by an analogous condition in which  $\Xi_{t,t+\tau}^d = \rho_{t,t+\tau}(P_t/P_{t+\tau})(Q_{t+\tau}^d + Q_{t+\tau}^x)(P_{t+\tau}^d)^{-\nu}$ ; the import price is assumed to be  $p_{t,t}^m = P_t^m = (\nu/(\nu-1))e_t P_t^*$  (this ensures immediate pass through of exchange rate changes into the import price).

<sup>16</sup> Under PCP,  $\mu_t^m = \nu/(\nu-1)$  and  $\mu_t^d = \mu_t^x$  hold. Thus,  $\mu_t^m = \mu_t^d = \mu_t^x = \nu/(\nu-1)$  under perfect domestic PPI inflation stabilization (the latter implies  $\mu_t^d = \nu/(\nu-1)$ , as discussed above), i.e. markup factors are identical to those in the flex-prices economy. This implies that the behavior of real variables is likewise the same as that under flexible prices.

<sup>17</sup> Previous studies on optimal monetary policy generally make the rather unrealistic assumption that the government uses producer subsidies (that are financed via lump-sum taxes) to off-set monopolistic distortions.

<sup>18</sup> Domestic firms take the export price index  $P_t^x$  as given, when setting their individual export prices; hence, these prices do not maximize total profits (of all domestic firms) in the export market. The household acts competitively in the international bond market, although its foreign asset position influences the interest rate  $i_t^f$ .

Table 4  
Historical and predicted standard deviations (in %)

	<i>Y</i>	<i>C</i>	$\Pi$	<i>i</i>	<i>e</i>	<i>RER</i>	<i>NFA</i>
Data: 1973–1994	1.52	1.45	0.70	0.46	9.13	8.89	12.72
Data: 1959–1970	1.51	1.32	0.83	0.39	0.46	0.98	<i>u</i>
Model: optimized policy rule	1.13	1.19	0.24	0.15	7.17	6.63	8.68
Model: exchange rate peg	1.85	2.25	0.64	0.46	0.00	0.72	4.96

Notes: *Y*: GDP; *C*: consumption;  $\Pi$ : gross CPI inflation; *i*: domestic interest rate; *e*: nominal exchange rate; *RER*: real exchange rate; *NFA*: net foreign assets; *u*: data unavailable.

The rows labeled “Data” show averages of historical standard deviations computed for Japan, Germany and the UK. The data are taken from International Financial Statistics and OECD Main Economic Indicators (see Kollmann, 2001a, for details), except the series on net foreign assets that were transcribed from Fig. 2 (series CUMCA) in Lane and Milesi-Ferretti (2001) [LMF]. The historical consumption series represents total private consumption; the nominal and real exchange rate series are bilateral exchange rates of the three countries vis-à-vis the US (the historical real exchange rate series are CPI based); the domestic interest rates are quarterly rates expressed in fractional units. The historical time series are quarterly, with the exception of net foreign assets that are annual. LMF provide net foreign assets series that have been normalized by annual GDP; I multiplied these net foreign assets series by 4 to facilitate the comparison with the theoretical net foreign assets variable considered in Tables 1–3 (there, net foreign assets are normalized by quarterly GDP).

The predicted standard deviations were generated using the baseline sticky-prices model. The variant with the optimized policy rule [row 3] assumes the four simultaneous shocks. The variant with the pegged exchange rate [row 4] assumes that there are no UIP shocks. The predicted statistic for net foreign assets reported in this Table pertains to a net foreign assets series sampled at an annual frequency (i.e., every four periods), normalized by annual GDP and multiplied by 4. The remaining theoretical statistics refer to quarterly variables.

Historical and predicted variables have been logged (with the exception of *i* and *NFA*) and HP filtered (the HP smoothing parameter was set at 400 [1600] for annual [quarterly] series). All statistics have been multiplied by 100, i.e. expressed in percentage terms.

rate rule targets GDP inflation. Under PTM, CPI targeting yields (essentially) the same welfare ( $\zeta = 0.39\%$ ) as domestic PPI inflation targeting ( $\zeta = 0.39\%$ ), but GDP inflation targeting results in lower welfare ( $\zeta = 0.24\%$ ). Under PTM, the GDP deflator is much more responsive to exchange rate movements than the CPI or the domestic PPI (the GDP deflator depends on export prices, expressed in producer currency,  $e_t p_t^x(s)$ ; under PTM, export prices are set in buyer currency, and  $e_t p_t^x(s)$  is thus highly responsive to exchange rate movements); hence, GDP inflation targeting generates a markedly more volatile nominal interest rate, and more volatile consumption than domestic PPI inflation targeting. (Under PCP, by contrast, GDP inflation targeting yields essentially the same welfare as domestic PPI inflation targeting, while CPI inflation targeting generates lower welfare.)

### 3.6. Standard deviations: comparing data and model predictions

Table 4 reports historical standard deviations that have been averaged across Japan, Germany and the U.K. (most standard deviations are quite similar across

these countries). Two sample periods are considered: 1959–1970 (Bretton Woods era, BW) and 1973–1994 (post-BW). The data are quarterly (net foreign assets are annual) and have been logged (with the exception of interest rates and net foreign assets) and HP filtered. The standard deviations of (HP filtered) nominal and real exchange rates have been markedly higher in the post-BW era (about 9%) than under BW (below 1%); by contrast, the standard deviations of the remaining variables have been fairly similar across these two eras (during both eras, the (average) standard deviation of GDP was about 1.5%; consumption, inflation, and the interest rate have been less volatile than GDP).

For the variant of the baseline sticky-prices model with the optimized policy rule and the variant with the pegged exchange rate (no UIP shocks in that version), Table 4 reports predicted standard deviations for logged and HP filtered variables (by contrast, the model predictions shown in Tables 1–3 pertain to variables that have neither been logged nor HP filtered). The predicted standard deviations generated by the two variants of the model are mostly similar to the historical statistics. Note in particular that the model captures the fact that both nominal and real exchange rate volatility have risen sharply after the end of the BW system, whereas the volatility of output has shown little change.

#### 4. Conclusions

This paper has computed welfare-maximizing Taylor-style interest rate rules, in a business cycle model of a small open economy with staggered price setting. Shocks to domestic productivity, to the world interest rate, to world inflation and to the uncovered interest rate parity condition are assumed. Optimized policy rules have a strict anti-inflation stance and imply significant nominal and real exchange rate volatility. The country responds to an increase in external volatility by holding more foreign assets. The policy rule affects the variances and the means of macrovariables, and this effect on the means matters significantly for welfare.

#### Appendix

**UIP shocks and biased exchange rate forecasts:** Assume that household *beliefs* at  $t$  about  $e_{t+1}$  are given by a probability density function,  $f_t^s$ , that differs from the true pdf,  $f_t$ , by a factor  $1/\varphi_t$ :  $f_t^s(e_{t+1}, \Omega) = f_t(e_{t+1}/\varphi_t, \Omega)/\varphi_t$ , where  $\Omega$  is any other random variable. The Euler equation for foreign currency bonds is then given by (20), i.e., the “bias parameter”  $\varphi_t$  corresponds to the “UIP shock” that appears in (20).

**Measurement of nominal/real GDP, GDP deflator:** In the model, nominal GDP equals the revenue of domestic intermediate goods producers:  $Y_t^{\text{nom}} = \int_0^1 [p_t^d(s)q_t^d(s) + e_t p_t^x(s)q_t^x(s)] ds$ . Evaluating  $q_t^d(s)$ ,  $q_t^x(s)$  at the prices of some baseline period gives *real* GDP. Here, baseline prices are normalized at unity (since all

domestic intermediate goods prices are identical in steady state). Thus  $Y_t = \int_0^1 q_t^d(s) + q_t^x(s) ds$ . The GDP deflator is  $Y_t^{\text{nom}}/Y_t$ .

**The solution method** (Sims, 2000): A technical note available from me shows that the aggregate dynamics of the economy is determined by a system of equations that can be written as

$$E_t G(\omega_{t+1}, \omega_t, \varepsilon_{t+1}) = 0, \tag{A.1}$$

where  $\omega_t$  is a vector of aggregate variables known at  $t$ , and  $\varepsilon_t = (\varepsilon_t^\theta, \varepsilon_t^\rho, \varepsilon_t^*, \varepsilon_t^i)'$ . The number of equations in (A.1) equals the number of elements of  $\omega_t$ . (Nominal variables are non-stationary in this economy; (A.1) is written in terms of nominal variables that have been “stationarized” through division by domestic price indices or the world price level.) Let  $dz_t = z_t - z$  ( $z$ : value of  $z_t$  in (deterministic) steady state given by  $G(\omega, \omega, 0) = 0$ ). Under conditions detailed by Sims (2000), there exists a unique, stationary, second-order accurate solution of (A.1) that has the following form:

$$\begin{aligned} ds_{h,t} &= F_{1h} + F_{2h}(ds'_{t-1} \varepsilon'_t)' + (ds'_{t-1} \varepsilon'_t)F_{3h}(ds'_{t-1} \varepsilon'_t)', \quad h = 1, \dots, H, \\ dx_{j,t} &= M_{1j} + ds'_t M_{2j} ds_t, \quad j = 1, \dots, J. \end{aligned} \tag{A.2}$$

$s_t = (s_{1,t}, \dots, s_{H,t})'$ ,  $x_t = (x_{1,t}, \dots, x_{J,t})'$  are column vectors (with  $H$  and  $J$  elements, where  $H + J$  is the number of equations in (A.1)) that are functions of  $\omega_t$ :

$$(s'_t \ x'_t)' = Z\omega_t. \tag{A.3}$$

$F_{1h}, F_{2h}, F_{3h}, M_{1j}, M_{2j}, Z$  are matrices/vectors ( $Z$  is non-singular). The intercepts in (A.2) are weighted sums of the variances of the exogenous variables. The remaining coefficients do not depend on these variances.

Eqs. (A.2) and (A.3) allow to compute  $E(d\omega_t)$  and  $E(d\omega_t d\omega'_t)$  (to get second-order accurate expressions, terms in  $ds_t, dx_t, \varepsilon_t$  that are of order greater than two and the intercepts in (A.2) can be neglected).

Computer code posted on Chris Sims’ web page was used to implement the method. I thoroughly checked the code; a corrected version is available from me. (Collard and Juillard (2001) and Schmitt-Grohé and Uribe (2001b) also develop solutions of dynamic models based on second-order expansions.)

**Estimation of  $\lambda$  (see (23)):** Up to a (log-)linear approximation, (16), (20), (23) imply  $\tilde{r}_t - \tilde{r}_t^* = -\lambda(B_{t+1}/P_t^*)/\chi + E_t \ln(RER_{t+1}/RER_t) + \varphi_t - 1$ , where  $\tilde{r}_t = i_t - E_t \ln(P_{t+1}/P_t)$  and  $\tilde{r}_t^* = i_t^* - E_t \ln(P_{t+1}^*/P_t^*)$  are expected domestic and world real interest rates, and  $RER_t = e_t P_t^*/P_t$  is the real exchange rate. Lane and Milesi-Ferretti (2001) fit this equation to a panel of 21 OECD economies, using annualized % interest rates and net foreign assets normalized by annual exports. Based on instrumental variables (allowing for country fixed-effects), estimates of about 3 are obtained for the coefficient of normalized net foreign assets (Table 7, Cols. 5–8). In terms of the relation between quarterly fractional interest rate differentials and net foreign assets normalized by quarterly exports, this implies a coefficient  $\lambda = 3/1600 \cong 0.0019$  (the value used in the simulations).

**The demand for foreign currency bonds:** As noted in Section 3.1, (20) implies  $1 = E\{\varphi_t(1 + i_t^f)q_{t+1}\}$ , where  $q_{t+1} \equiv \beta(RER_{t+1}/RER_t)(C_t/C_{t+1})/\Pi_{t+1}^*$  is the household's marginal rate of substitution between units of the foreign currency available at  $t$  and  $t + 1$ . Substituting (23) into that equation and taking a second-order Taylor expansion gives

$$E\tilde{B}_{t+1} \cong \chi(\lambda\beta)^{-1}[E\hat{q}_{t+1} + (\beta/\Pi^*)Cov(i_t^f, \hat{q}_{t+1} + \varphi_t) + Cov(\varphi_t, \hat{q}_{t+1})], \tag{A.4}$$

and

$$E\hat{q}_{t+1} \cong \frac{1}{2}[Var(\hat{q}_{t+1}) + Var(\Pi_{t+1}^*)] > 0, \tag{A.5}$$

where  $\tilde{B}_{t+1} = B_{t+1}/P_t^*$ ,  $Cov(x, z)$ : covariance between  $x$  and  $z$ .

Thus  $E\hat{q}_{t+1}$  is higher in the stochastic economy than in the steady state. Furthermore  $E\hat{q}_{t+1}$  is increasing in  $Var(\hat{q}_{t+1})$ . The covariances in (A.4) are negative. (Positive world interest rate and UIP shocks lower the expected rate of exchange rate depreciation; thus,  $Cov(i_t^f, \hat{q}_{t+1} + \varphi_t) < 0$ ,  $Cov(\varphi_t, \hat{q}_{t+1}) < 0$ .) However, these terms are dominated by  $E\hat{q}_{t+1}$ , since  $\hat{q}_{t+1}$  is markedly more volatile than  $i_t^f$  and  $\varphi_t$  (e.g.,  $|Cov(i_t^f, \hat{q}_{t+1} + \varphi_t)| \ll Var(\hat{q}_{t+1})$ ). Thus,  $E\tilde{B}_{t+1} > 0$ . UIP shocks are the main source of real exchange rate fluctuations; hence, UIP shocks have a stronger effect on  $E\hat{q}_{t+1}$  and  $E\tilde{B}_{t+1}$  than the other types of shocks (in the baseline model with just UIP shocks,  $E\hat{q}_{t+1} = 0.25\%$ ; with the four simultaneous shocks,  $E\hat{q}_{t+1} = 0.28\%$ ).

**Price setting in the intermediate goods sector:** A domestic firm that gets to choose a new domestic price at date  $t$  sets that price at (see (8)):

$$p_{t,t}^d = (v/(v - 1)) \left\{ \sum_{\tau=0}^{\tau=\infty} \lambda_{t,t+\tau} E_t MC_{t+\tau} / \Pi^\tau \right\} + (v/(v - 1)) \frac{\sum_{\tau=0}^{\tau=\infty} (d\Pi^{1-v})^\tau Cov_t(\Xi_{t,t+\tau}^d, MC_{t+\tau} / \Pi^\tau)}{\sum_{\tau=0}^{\tau=\infty} (d\Pi^{1-v})^\tau E_t \Xi_{t,t+\tau}^d},$$

where

$$\lambda_{t,t+\tau} = (d\Pi^{1-v})^\tau E_t \Xi_{t,t+\tau}^d / \left\{ \sum_{\tau=0}^{\tau=\infty} (d\Pi^{1-v})^\tau E_t \Xi_{t,t+\tau}^d \right\} \quad \text{with} \quad \sum_{\tau=0}^{\tau=\infty} \lambda_{t,t+\tau} = 1.$$

( $Cov_t(x, y) \equiv E_t xy - E_t x E_t y$ : conditional covariance between  $x$  and  $y$ .) Hence,  $p_{t,t}^d$  equals a weighted average of expected future (detrended) marginal costs (multiplied by the steady state markup factor  $v/(v - 1)$ ), plus a weighted sum of (conditional) covariances between future marginal costs and  $\Xi_{t,t+\tau}^d = \beta^\tau (C_t/C_{t+\tau})(P_t/P_{t+\tau})Q_{t+\tau}^d (P_{t+\tau}^d)^v$ .  $Cov_t(\Xi_{t,t+\tau}^d, MC_{t+\tau} / \Pi^\tau)$  (and, thus,  $p_{t,t}^d$ ) is higher, the higher the covariance between  $MC_{t+\tau} / \Pi^\tau$  and date  $t + \tau$  demand for the product sold by the firm (that demand is proportional to  $Q_{t+\tau}^d (P_{t+\tau}^d)^v$ ; see (2)).



## References

- Aoki, K., 2001. Optimal policy responses to relative-price changes. *Journal of Monetary Economics* 48, 55–80.
- Bacchetta, P., van Wincoop, E., 2000. Does exchange rate stability increase trade and welfare? *American Economic Review* 90, 1093–1109.
- Backus, D., Kehoe, P., Kydland, F., 1995. International business cycles: theory and evidence. In: Cooley, T. (Ed.), *Frontiers of Business Cycle Research*. Princeton University Press, Princeton, pp. 331–356.
- Batini, N., Nelson, E., 2000. When the bubble bursts: monetary policy rules and foreign exchange market behavior. Working Paper, Bank of England.
- Batini, N., Harrison, R., Millard, S., 2001. Monetary policy rules for an open economy. Working Paper, Bank of England.
- Baxter, M., Stockman, A., 1989. Business cycles and the exchange rate regime. *Journal of Monetary Economics* 23, 377–400.
- Benigno, G., 1999. Real exchange rate persistence and monetary policy rules. Working Paper, Bank of England.
- Benigno, P., 2000. Optimal monetary policy in a currency area. Working Paper, New York University.
- Benigno, P., 2001. Price stability with imperfect financial integration. Working Paper, New York University.
- Bergin, P., 2001. Putting the “new international macroeconomics” to a test. Working Paper, University of California, Davis.
- Betts, C., Devereux, M., 2001. The international effects of monetary and fiscal policy in a two-country model. In: Calvo, G., Dornbusch, R., Obstfeld, M. (Eds.), *Money Capital Mobility and Trade: Essays in Honor of Robert Mundell*. MIT Press, Cambridge, MA, pp. 9–52.
- Calvo, G., 1983. Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12, 383–398.
- Campa, J., Goldberg, L., 2001. Exchange rate pass-through into import prices: a macro and micro phenomenon. Working Paper, Federal Reserve Bank of New York.
- Chari, V., Kehoe, P., McGrattan, E., 2000. Can sticky price models generate volatile and persistent real exchange rates? Research Department Staff Report 223, Federal Reserve Bank of Minneapolis.
- Clarida, R., Galí, J., Gertler, M., 1999. The science of monetary policy: a new Keynesian perspective. *Journal of Economic Literature* 37, 1661–1707.
- Clarida, R., Galí, J., Gertler, M., 2001. Optimal monetary policy in open versus closed economies. *American Economic Review, AEA Papers and Proceedings* 91, 248–252.
- Collard, F., Dellas, H., 2002. Exchange rate systems and macroeconomic stability. *Journal of Monetary Economics* 49, 571–599.
- Collard, F., Juillard, M., 2001. Perturbation methods for rational expectations models. Working Paper, GREMAQ and CEPREMAP.
- Corsetti, G., Pesenti, P., 2001. International dimensions of optimal monetary policy. Working Paper No. 7665, National Bureau of Economic Research.
- Dedola, L., Leduc, S., 2001. Why is the business cycle behavior of fundamentals alike across exchange rate Regimes? Working Paper, Banca d’Italia and Federal Reserve Bank of Philadelphia.
- Devereux, M., Engel, C., 2000. Monetary policy in the open economy revisited: price setting and exchange rate flexibility. Working Paper No. 7665, National Bureau of Economic Research.
- Duarte, M., Stockman, A., 2001. Rational speculation and exchange rates. Working Paper, Federal Reserve Bank of Richmond and University of Rochester.
- Erceg, A., Levin, A., 2001. The effects of dollarization on macroeconomic stability. Working Paper, Board of Governors of the Federal Reserve System.
- Erceg, C., Henderson, D., Levin, A., 2000. Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics* 46, 281–313.
- Faia, E., 2001. Stabilization policy in a two country model and the role of financial frictions. Working Paper No. 56, European Central Bank.

- Frankel, J., Froot, K., 1989. Forward discount bias: is it an exchange risk premium? *Quarterly Journal of Economics* 104, 139–161.
- Gali, J., Monacelli, T., 2000. Optimal monetary policy and exchange rate volatility in a small open economy. Working Paper, Universitat Pompeu Fabra and Boston College.
- Ghironi, F., Rebucci, A., 2001. Monetary rules for emerging market economies. Working Paper, Boston College.
- Hairault, J., Patureau, L., Sopraseuth, T., 2001. Overshooting and the exchange rate disconnect puzzle. Working Paper, University of Paris I.
- Hooper, P., Marquez, J., 1995. Exchange rates, prices, and external adjustment in the United States and Japan. In: Kenen, P. (Ed.), *Understanding Interdependence*. Princeton University Press, Princeton, pp. 107–168.
- Ireland, P., 1996. The role of countercyclical monetary policy. *Journal of Political Economy* 104, 704–723.
- Jeanne, O., Rose, A., 2000. Noise trading and exchange rate regimes. Working Paper, University of California, Berkeley.
- Judd, K., 1998. *Numerical Methods in Economics*. MIT Press, Cambridge, MA.
- Knetter, M., 1993. International comparison of pricing to market behavior. *American Economic Review* 83, 473–486.
- Kollmann, R., 1995. Consumption, real exchange rates and the structure of international asset markets. *Journal of International Money and Finance* 14, 191–211.
- Kollmann, R., 1996. Incomplete asset markets and the cross-country consumption correlation puzzle. *Journal of Economic Dynamics and Control* 20, 945–961.
- Kollmann, R., 2001a. The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation. *Journal of International Economics* 55, 243–262.
- Kollmann, R., 2001b. Explaining international comovements of output and asset returns: the role of money and nominal rigidities. *Journal of Economic Dynamics and Control* 25, 1547–1583.
- Kollmann, R., 2002. Macroeconomic effects of nominal exchange rate regimes: new insights into the role of price dynamics. Working Paper, University of Bonn.
- Lane, P., 2001. The new open economy macroeconomics: a survey. *Journal of International Economics* 54, 235–266.
- Lane, P., Milesi-Ferretti, G., 2001. Long term capital movements. Working Paper, Trinity College Dublin and IMF.
- Lewis, K., 1995. Puzzles in international financial markets. In: Grossman, G., Rogoff, K. (Eds.), *Handbook of International Economics*, Vol. 3. Elsevier, Amsterdam, pp. 1913–1971.
- Lopez-Salido, D., 2000. Work in progress, Bank of Spain.
- Mark, N., Wu, Y., 1998. Rethinking deviations from uncovered interest parity: the role of covariance risk and noise. *Economic Journal* 108, 1686–1706.
- Martins, J., Scarpetta, S., Pilat, D., 1996. Mark-up pricing, market structure and the business cycle. *OECD Economic Studies* 27, 71–105.
- McCallum, B., Nelson, E., 1999. Nominal income targeting in an open-economy optimizing model. *Journal of Monetary Economics* 43, 554–578.
- McCallum, B., Nelson, E., 2000. Monetary policy for an open economy: an alternative framework with optimizing agents and sticky prices. *Oxford Review of Economic Policy* 16, 74–91.
- Monacelli, T., 1999. Into the Mussa puzzle: monetary policy regimes and the real exchange rate in a small open economy. Working Paper, Boston College.
- Obstfeld, M., Rogoff, K., 2000. New directions for stochastic open economy models. *Journal of International Economics* 50, 117–153.
- Parrado, E., Velasco, A., 2001. Optimal interest rate policy in a small open economy: the case for a clean float. Working Paper, NYU.
- Rogoff, K., 2000. Monetary models of dollar/yen/euro nominal exchange rates: dead or undead? Working Paper, Harvard University.
- Rotemberg, J., Woodford, M., 1997. An optimization-based econometric framework for the evaluation of monetary policy. *NBER Macroeconomics Annual* 12, 297–346.
- Schmitt-Grohé, S., Uribe, M., 2001a. Stabilization policy and the costs of dollarization. *Journal of Money Credit and Banking* 33, 482–509.

- Schmitt-Grohé, S., Uribe, M., 2001b. Solving dynamic general equilibrium models using a second-order approximation of the policy function. Working Paper, Rutgers University and University of Pennsylvania.
- Senhadji, A., 1997. Sources of debt accumulation in a small open Economy. Working Paper 97/146, International Monetary Fund.
- Sims, C., 2000. Second order accurate solution of discrete time dynamic equilibrium models. Working paper, Economics Department, Princeton University ([www.princeton.edu/~sims](http://www.princeton.edu/~sims)).
- Smets, F., Wouters, R., 2000. Optimal monetary policy in an open economy: an application to the Euro area. Working Paper, European Central Bank.
- Smets, F., Wouters, R., 2001. Openness, imperfect exchange Rate pass-through and monetary policy. Working Paper, European Central Bank.
- Sutherland, A., 2001. Incomplete pass-through and the welfare effects of exchange rate variability. Working Paper, University of St. Andrews.
- Taylor, J., 1993a. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy 39, 195–214.
- Taylor, J., 1993b. *Macroeconomic Policy in a World Economy*. Norton, New York.
- Taylor, J., 1999. *Monetary Policy Rules*. University of Chicago Press, Chicago.
- Yun, T., 1996. Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics* 37, 345–370.