

## Consumption, real exchange rates and the structure of international asset markets

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International Real Business Cycle (IRBC) models that assume complete asset markets yield strong restrictions for consumption and bilateral real exchange rate series. Empirical tests with data for the US, Japan, France, UK, Italy, Canada and Sweden suggest that neither the trend behavior nor high-frequency movements of consumption and real exchange rates are well explained by IRBC models with complete markets. It appears, however, that high-frequency fluctuations of consumption and real exchange rates are consistent with unrestricted international trade in risk-free bonds. (JEL F41).

Recent research has extended the closed-economy Real Business Cycle (RBC) model (Kydland and Prescott, 1982; King *et al.*, 1988) to an international setting. In International Real Business Cycle (IRBC) models, international transactions take place in goods markets as well as in asset markets (see, *eg*, Backus *et al.*, 1992; Baxter and Crucini, 1993).<sup>1</sup> These models typically assume complete international asset markets. IRBC models with complete asset markets yield strong restrictions for consumption and bilateral real exchange rate series because complete markets make possible extensive international risk-sharing.

This paper shows that with time-separable and iso-elastic preferences (as typically assumed in IRBC theory) and under certain restrictions on taste shocks, models with complete markets imply that logged consumption and bilateral real exchange rate series should be cointegrated for any pair of countries. The empirical

\* This paper is based on a chapter of my Ph.D. dissertation at the University of Chicago. I am grateful to John Huizinga and Michael Woodford for advice. Thanks for useful suggestions are also extended to Bryan Campbell, Fabio Canova, Chris Erceg, Christophe Faugère, René Garcia, John Heaton, Gregor Smith, Pierre Perron, to an anonymous referee and to seminar participants at Chicago, Queen's, Montréal, the 1992 Econometric Society European Meetings (Brussels) and the 1992 Canadian Macroeconomics Study Group (London, Ontario). Financial support from the Bradley Foundation, the Sloan Foundation (through a grant to the International Economics workshop at the University of Chicago) as well as from Fonds Marcel Fairbault and Fonds CAFIR (Université de Montréal) is gratefully acknowledged.

evidence presented in this paper rejects this theoretical prediction for a sample of countries consisting of the USA, Japan, France, UK, Italy, Canada and Sweden.

The risk-sharing made possible by complete markets also implies that, for any pair of countries, there is a close relationship between the growth rates of consumption in these countries and the growth rate of their bilateral real exchange rate. The paper casts doubts on the empirical validity of this relation between high-frequency consumption and real exchange rate movements as well.

As an alternative to complete asset markets models, recent IRBC research has started to investigate models of the international economy with incomplete asset markets. For example, Kollmann (1990) and Baxter and Crucini (1991) present two-country IRBC models in which only real risk-free bonds can be used for international financial transactions.<sup>2</sup> Unrestricted trade in real risk-free bonds implies that expected intertemporal marginal rates of substitution in consumption are equated across countries. Tests based on the Generalized Method of Moments fail to reject this prediction.

The tests of complete markets IRBC models that are presented in this paper build on work by Leme (1984) who has examined international consumption data in order to study international risk-sharing, but without presenting formal statistical tests and without taking into consideration the implications of fluctuations in real exchange rates for international risk-sharing.<sup>3</sup> The present paper is also related to work by Neusser (1991) who has previously used cointegration methods to test (closed and open economy) RBC models. However Neusser's analysis also leaves out the real exchange rate. The finding that high-frequency consumption and real exchange rate movements are consistent with fully integrated markets for real risk-free bonds is closely related to empirical results presented by Obstfeld (1989) which suggest that (at least for the period after 1972) the behavior of consumption series for the USA, Japan and Germany is largely consistent with free international trade in *nominal* risk-free bonds.

Section I discusses testable implications of IRBC models with complete international asset markets for the behavior of consumption and real exchange rate series. Section II describes the unit root/cointegration tests used in this paper. Section III describes the data, while Section IV presents empirical tests of the complete markets framework. Section V tests a restriction on consumption and real exchange rate series that holds with free international trade in risk-free real bonds. Section VI summarizes the paper.

## I. Complete international asset markets and the behavior of consumption and real exchange rates

A world with  $K$  countries indexed by  $k = 1, \dots, K$  is considered. Each country is inhabited by an infinitely lived representative agent. Following the existing IRBC literature, it is assumed that the intertemporal preferences of country  $k$  are represented by a time-separable utility function of the form  $E_s \{ \sum_{t=s}^{\infty} (\beta^k)^{t-s} u_t^k(c_t^k) \}$ . Here,  $E_s$  denotes expectations conditional on information available in period  $s$  (all countries have access to the same information).  $0 < \beta^k < 1$  is country  $k$ 's subjective discount factor and  $u_t^k(c_t^k)$  is  $k$ 's instantaneous utility

function in period  $t$ . This function is assumed to be increasing and strictly concave.  $c_t^k$  is an aggregate consumption good that represents the basket of commodities consumed by country  $k$ .

Many IRBC models assume a world with a single consumption good that is consumed by all countries (see, *eg*, Backus *et al.*, 1992; Baxter and Crucini, 1993).<sup>4</sup> In such a world, bilateral real exchange rates are constant. The large variations in real exchange rates observed in the data make it difficult to justify a single-good model of the world economy. This is why, in the following analysis, the goods consumed by different countries are allowed to differ.

Given the preferences just described, the existence of complete international asset markets implies that, in equilibrium, the following condition is satisfied for any country pair  $i, j$  and for all dates and states of the world (see the Appendix for a derivation):

$$\langle 1 \rangle \quad (\beta^i)^t u_t^i = \Lambda^{i,j} (\beta^j)^t u_t^j R_t^{i,j}.$$

Here,  $u_t^i$  and  $u_t^j$  are the marginal utilities of consumption for countries  $i$  and  $j$  in period  $t$ .  $\Lambda^{i,j}$  is a term that is time-invariant (and invariant across states of the world), while  $R_t^{i,j}$  is the date  $t$  price of one unit of country  $i$ 's consumption good in terms of country  $j$ 's consumption good—*ie*, the bilateral real exchange rate between the two countries (in terms of their respective aggregate consumption goods). Because of the assumed strict concavity of  $u_t^i$  and  $u_t^j$ , this fundamental risk-sharing condition implies that—holding constant the bilateral real exchange rate between countries  $i$  and  $j$ —country  $j$ 's consumption can be expressed as an increasing function of country  $i$ 's consumption.

To obtain testable implications from  $\langle 1 \rangle$ , I follow the IRBC literature and assume an iso-elastic instantaneous period utility function:<sup>5</sup>

$$\langle 2 \rangle \quad u_t^k(c) = A_t^k (1/\sigma^k) c^{\sigma^k}, \quad \text{with } A_t^k > 0, \sigma^k < 1.$$

The term  $A_t^k$  represents a stochastic taste shock.<sup>6</sup>

When the preferences specified in  $\langle 2 \rangle$  are assumed, the risk-sharing condition  $\langle 1 \rangle$  yields the following restriction on logged consumption in countries  $i$  and  $j$  and on the logged real exchange rate between these countries:

$$\langle 3 \rangle \quad (\sigma^i - 1) \ln(c_t^i) = K_t^{i,j} + \ln(\beta^j/\beta^i)t + (\sigma^j - 1) \ln(c_t^j) + \ln(R_t^{i,j}),$$

where  $K_t^{i,j} \equiv \ln(\Lambda^{i,j} A_t^j/A_t^i)$ .

#### *I.A. Complete asset markets and the trend behavior of consumption and real exchange rates*

Statistical tests discussed below suggest that logged consumption and bilateral real exchange rate series in the sample of countries considered in this paper follow unit-root processes. Under the assumption that the term  $K_t^{i,j}$  is trend-stationary (*ie*, that it can be represented as the sum of a deterministic trend and a covariance-stationary random variable), the risk-sharing condition  $\langle 3 \rangle$  implies that  $\ln(c_t^i)$ ,  $\ln(c_t^j)$  and  $\ln(R_t^{i,j})$  are cointegrated, *ie*, that there exists a linear combination of these variables that is trend-stationary.

In a world with a single consumption good, the risk-sharing condition  $\langle 3 \rangle$

implies that logged consumption is cointegrated across countries (as bilateral real exchange rates are constant in such a world). Neusser's (1991) finding that, in a sample of OECD countries, consumption fails to be driven by just one common stochastic trend casts doubts on this prediction. However, as discussed earlier, the strong variations in real exchange rates that are observed empirically make it difficult to justify a single-good model. It seems important to point out that—up to a log-linear approximation—linear cointegrating relationships similar to <3> follow from the risk-sharing condition <1> for functional forms of the period utility function that are more general than <2>. Note also that the RBC literature typically assumes utility functions in which consumption and work effort interact in a non-separable way; this literature assumes, however, that per capita hours of work are covariance-stationary (see, for example, King *et al.*, 1988). Under this assumption, logged consumption and bilateral real exchange rate series continue to be cointegrated when international asset markets are complete.<sup>7</sup>

Section II describes the unit root and cointegration tests that will be used to test the restrictions on the trend behavior of consumption and real exchange rates just discussed.

### *I.B. Complete asset markets and high-frequency movements in consumption and real exchange rates*

To study the high-frequency implications of complete markets, we take first differences of <3> which gives:

$$\langle 4 \rangle \quad (\sigma^i - 1)\Delta \ln(c_t^i) = \Delta K_t^{i,j} + \ln(\beta^j/\beta^i) + (\sigma^j - 1)\Delta \ln(c_t^j) + \Delta \ln(R_t^{i,j}),$$

where  $\Delta$  is the first-difference operator ( $\Delta x_t \equiv x_t - x_{t-1}$ ). Equation <4> cannot be tested unless strong assumptions about taste shocks are made. When we assume that there are no taste shocks (*ie*,  $\Delta K_t^{i,j} = 0$ ), or that taste shifts are small, then <4> can be tested by regressing any of the three variables  $\Delta \ln(c_t^i)$ ,  $\Delta \ln(c_t^j)$  and  $\Delta \ln(R_t^{i,j})$  on the remaining two variables (and on a constant). These regressions are then predicted to yield a very good fit of the data.

## **II. Unit root and cointegration tests**

The Augmented Dickey–Fuller (ADF) test is used to test the null hypothesis that the time series used in this study follow unit root processes. Two approaches for testing for cointegration are used: the ‘spurious regression’ method proposed by Park (1990, 1992) and two of the ‘residual-based’ cointegration tests presented by Phillips and Ouliaris (1990). Park’s method allows us to test the null hypothesis that a set of variables is cointegrated. This is an attractive feature since the model to be tested predicts that certain variables are cointegrated. In contrast, the Phillips and Ouliaris tests set up the null hypothesis that a set of variables is *not* cointegrated.

Assume that the  $q + 1$  variables  $x_t^0, x_t^1, \dots, x_t^q$  variables all have a unit root. The cointegration tests exploit the fact that if there exists a cointegrating

relationship between one of the variables, say  $x_t^0$ , and the remaining variables, then the residual in the following regression equation is covariance-stationary:

$$\langle 5 \rangle \quad x_t^0 = D + Ft + \sum_{s=1}^{s=q} \psi_s x_t^s + \eta_t.$$

The Park and the Phillips and Ouliaris tests exploit this fact. Park (1992) shows how to conduct tests of hypotheses concerning the  $\psi_s$  coefficients on the right-hand side of  $\langle 5 \rangle$  (under the null of cointegration). This is useful because the coefficients of the risk-sharing condition  $\langle 3 \rangle$  are functions of the preference parameters  $\sigma^i$  and  $\sigma^j$ . Versions of the regression equation  $\langle 5 \rangle$  that use  $\ln(c_t^i)$ ,  $\ln(c_t^j)$  and  $\ln(R_t^{i,j})$  for a given country pair  $i-j$  as 'x' variables can thus be used to estimate these preference parameters. The methods described in Park (1992) allow us to test whether the preference parameters are consistent with well-behaved utility functions.

### III. The data

Three data sets are used:

- (i) For the USA, Japan, France, UK, Italy and Canada, quarterly data on private non-durables plus services consumption expenditures for the period 1971:I–1988:I were obtained from OECD *Quarterly National Accounts* (QNA).
- (ii) For the USA, Japan, UK and Canada, quarterly series on total private consumption expenditures for the period 1957:I–1991:II were taken from the *International Financial Statistics* (IFS) database compiled by the IMF.
- (iii) For the USA, UK and Sweden, annual historical private consumption series for the period 1889/1920–1990 were compiled from Backus and Kehoe (1990) and from the IFS.

All consumption data are seasonally adjusted, in constant prices and in per capita terms (using consumption series that are not in per capita form does not significantly affect the results). For additional information on the data, see the Appendix.

### IV. Empirical results for the complete markets model

#### IV.A. Results from unit root tests

Table 1 tests for a unit root in logged private non-durables plus services consumption expenditures (from OECD QNA) for the USA, Japan, France, UK, Italy and Canada, as well as in the logged bilateral real exchange rates for pairs of these countries.

Overall, the table yields little evidence at the 10 percent level against the unit root hypothesis.<sup>8</sup> The same finding obtains for the IFS consumption series, the historical consumption series, as well as for the corresponding bilateral real exchange rate series (to save space, the test results for these series are not presented in Table 1).

TABLE 1. Augmented Dickey-Fuller unit root tests.

(a) Logged non-durables plus services consumption (from the OECD QNA)						
k = 0	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
USA	-1.36	-1.98	-3.32 #	-2.97	-2.23	-2.33
JA	-3.26 #	-3.07	-4.79 **	-4.44 **	-3.66 *	-3.13
FR	-3.28 #	-2.86	-2.38	-2.29	-2.47	-2.29
UK	0.41	-0.08	-0.38	-0.29	-1.12	-1.18
IT	-1.38	-3.33 #	-2.11	-2.78	-2.08	-2.48
CA	-2.32	-2.01	-2.00	-1.99	-2.34	-2.86
(b) Logged bilateral real exchange rates (in terms of non-durables + services)						
k = 0	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
USA-JA	-1.43	-2.23	-2.00	-1.70	-1.71	-1.56
USA-FR	-1.50	-2.09	-2.07	-1.89	-2.32	-2.32
USA-UK	-1.49	-1.83	-1.88	-2.04	-1.73	-2.38
USA-IT	-0.75	-1.83	-2.18	-1.86	-2.17	-2.12
USA-CA	-1.40	-2.08	-2.51	-2.50	-2.31	-2.24
JA-FR	-2.07	-3.36 #	-3.76 *	-2.76	-2.56	-2.65
JA-UK	-1.45	-2.34	-2.30	-1.98	-1.83	-1.71
JA-IT	-2.08	-3.29 #	-3.47 #	-2.41	-2.14	-2.05
JA-CA	-1.47	-2.34	-2.20	-1.82	-2.08	-1.88
FR-UK	-1.85	-2.02	-2.24	-2.12	-2.13	-2.21
FR-IT	-1.67	-1.90	-2.00	-1.65	-1.98	-2.02
FR-CA	-1.45	-2.17	-2.16	-1.91	-2.52	-2.50
UK-IT	-1.21	-1.48	-1.19	-1.51	-0.91	-1.24
UK-CA	-1.39	-1.84	-1.94	-2.07	-1.88	-2.39
IT-CA	-1.07	-2.20	-2.53	-2.09	-2.56	-2.58

Notes: The table reports Augmented Dickey-Fuller test statistics.  $k$ : the number of lagged  $\Delta x$  terms included on the right-hand side of equation  $\langle A1 \rangle$  in the Appendix. The sample period is 1971:III-1988:I for  $k = 0$ ; for  $k > 0$ , the beginning of the sample period is shifted forward (eg, for  $k = 1$  the sample period is 1971:III-1988:I).

\*, \* and #: rejection of the unit root hypothesis at 1%, 5% and 10% significance levels respectively. For 50 observations, the critical values at these levels are -4.15, -3.50 and -3.18 respectively (see Table 8.5.2 in Fuller, 1976).  
 JA: Japan, FR: France, IT: Italy, CA: Canada.

TABLE 2. Probability values for Park test of null hypothesis of cointegration.

(a) Quarterly non-durables plus services consumption (OECD QNA, 1971:I–1988:I)			
USA–JA	0.69 (P)	0.02 (P)	0.05 (P)
USA–FR	0.61	0.27 (P)	0.03 (P)
USA–UK	0.63	0.03	0.09
USA–IT	0.35 (P)	0.21 (P)	0.03 (P)
USA–CA	0.26 (P)	0.02 (P)	0.13 (P)
JA–FR	0.03	0.16	0.16
JA–UK	0.03	0.02 (P)	0.11 (P)
JA–IT	0.02 (P)	0.11	0.27 (P)
JA–CA	0.04	0.29	0.73
FR–UK	0.54 (P)	0.08 (P)	0.09 (P)
FR–IT	0.23 (P)	0.72	0.09 (P)
FR–CA	0.82 (P)	0.11	0.24 (P)
UK–IT	0.04	0.06	0.13
UK–CA	0.05	0.02	0.11 (P)
IT–CA	0.69	0.10	0.03 (P)
(b) Quarterly total consumption (IFS, 1957:I–1991:II)			
USA–JA	0.22	0.00	0.45
USA–UK	0.01 (P)	0.08 (P)	0.06 (P)
USA–CA	0.07	0.04	0.01
JA–UK	0.00 (P)	0.12 (P)	0.55 (P)
JA–CA	0.02	0.02	0.16
UK–CA	0.07 (P)	0.01 (P)	0.03 (P)
(c) Historical data (annual) on total consumption (1889–1990/1920–90)			
USA–UK	0.20 (P)	0.08 (P)	0.10 (P)
USA–SW	0.03 (P)	0.03 (P)	0.25 (P)
UK–SW	0.03	0.00 (P)	0.19 (P)

Notes: The table reports *p*-values for Park tests of the null hypothesis of cointegration.

Let *i* and *j* denote the first and the second country listed for a given country pair respectively (eg, for USA–JA, *i* = USA and *j* = Japan). The 1st, 2nd and 3rd test statistics reported for country pair *i*–*j* pertain to versions of the regression equation <5> that use  $\ln(c_t^i)$ ,  $\ln(c_t^j)$  and  $\ln(R_t^{i,j})$  respectively as left-hand side variables.

A 'P' reported next to the 1st, 2nd or 3rd test statistic for a given country pair indicates that for at least one of the countries in that pair, the preference parameters recovered from the corresponding version of the regression equation <5> violate the concavity restriction  $\sigma < 1$  and that one cannot reject the hypothesis (at the 5% level) that  $\sigma = 1$ .

The sample period in panel (c) is 1889–1990 for USA–UK and 1920–90 for USA–SW and UK–SW.

JA: Japan, FR: France, It: Italy, CA: Canada, SW: Sweden.

## IV.B. Results from cointegration tests

For all country pairs in the three data sets, tests were conducted to see whether  $\ln(c_t^i)$ ,  $\ln(c_t^j)$  and  $\ln(R_t^{i,j})$  are cointegrated. Table 2 reports  $p$ -values for Park tests of the null of cointegration, while Table 3 gives the Phillips and Ouliaris (1990)  $\hat{Z}_\alpha$  and  $\hat{Z}_t$  test statistics for the null of no cointegration (the  $\hat{Z}_\alpha$  and  $\hat{Z}_t$  statistics are shown in columns 1–3 and in columns 4–6 of Table 3 respectively). Table 4 summarizes the outcome of the cointegration tests.

In finite samples, the outcome of the cointegration tests can depend on which of the 'x' variables is selected as the dependent variable in the regression equation <5>. Hence, test results for all possible choices concerning the dependent variable are reported.

*Results for OECD data on non-durables plus services consumption expenditures*

A total of 45 Park test statistics were computed for the OECD data set (as there are 15 country pairs in that data set). The Park test rejects the null of cointegration at the 10 percent level for 20 of these 45 test statistics. It appears that the preference parameters estimated using the Park method frequently violate concavity: among the 25 cases where the null of cointegration fails to be rejected, there are 13 cases where statistically significant (at the 5 percent level) violations of concavity occur. Taken together, these findings cast serious doubts on the model. The same conclusion emerges from the Phillips and Ouliaris tests: the no-cointegration

TABLE 3. Phillips and Ouliaris tests of null hypothesis of cointegration.

(a) Quarterly non-durables + services consumption (OECD QNA, 1971:II-1988:I)						
	$\hat{Z}_\alpha$			$\hat{Z}_t$		
	(1)	(2)	(3)	(4)	(5)	(6)
US-JA	-11.1	-11.3	-9.7	-2.8	-2.4	-2.0
US-FR	-43.7**	-13.4	-22.4	-5.5**	-2.6	-3.5
US-UK	-5.0	-9.2	-7.2	-1.1	-2.1	-1.9
US-IT	-5.3	-9.5	-8.0	-1.5	-2.1	-1.7
US-CA	-8.0	-8.8	-9.2	-2.3	-2.0	-2.0
JA-FR	-24.5	-13.6	-11.6	-3.7	-3.0	-2.5
JA-UK	-6.0	-11.0	-8.3	-1.2	-2.9	-2.1
JA-IT	-11.2	-14.7	-8.0	-2.5	-3.2	-2.0
JA-CA	-11.2	-11.4	-11.4	-2.4	-2.5	-2.4
FR-UK	-5.3	-25.2	-11.4	-1.2	-3.8	-2.4
FR-IT	-8.9	-36.4*	-18.0	-2.1	-4.7**	-3.1
FR-CA	-12.2	-36.7*	-16.4	-2.8	-5.1**	-3.0
UK-IT	-7.2	-10.1	-12.6	-1.9	-1.8	-2.5
UK-CA	-8.2	-4.2	-8.6	-2.5	-0.9	-2.1
IT-CA	-9.4	-8.1	-8.6	-2.6	-2.0	-2.0

*continued*



TABLE 3. *Continued*

(b) Quarterly total consumption (IFS, 1957:II–1991:II)						
	$\hat{Z}_x$			$\hat{Z}_i$		
	(1)	(2)	(3)	(4)	(5)	(6)
USA–JA	–15.3	–11.9	–12.5	–2.8	–2.8	–2.5
USA–UK	–8.4	–28.8#	–18.3	–1.8	–3.9#	–3.0
USA–CA	–9.3	–6.0	–6.6	–2.2	–1.4	–1.3
JA–UK	–9.2	–37.0*	–26.3	–2.3	–4.6*	–3.7
JA–CA	–9.4	–13.3	–17.6	–2.8	–2.7	–2.9
UK–CA	–33.0*	–5.8	–17.6	–4.2*	–1.3	–2.8

(c) Historical data (annual) on total consumption (1889–1990/1920–90)						
	$\hat{Z}_x$			$\hat{Z}_i$		
	(1)	(2)	(3)	(4)	(5)	(6)
US–UK	–19.7	–8.6	–16.4	–3.3	–2.1	–2.7
US–SW	–14.6	–11.9	–11.1	–2.8	–2.5	–2.3
UK–SW	–5.6	–11.2	–10.9	–1.5	–2.4	–2.4

Notes: Columns 1–3 and columns 4–6 report the Phillips and Ouliaris  $\hat{Z}_x$  and  $\hat{Z}_i$  test statistics respectively. Let  $i$  and  $j$  denote the first and the second country listed for a given country pair respectively (eg. for USA–JA,  $i$  = USA and  $j$  = Japan). The 1st, 2nd and 3rd Phillips and Ouliaris (1990)  $\hat{Z}_x$  and  $\hat{Z}_i$  statistics reported for country pair  $i$ – $j$  use  $\ln(c^i)$ ,  $\ln(c^j)$  and  $\ln(R^{i,j})$  respectively as left-hand side variables in equation <5>.

\*\*, \*, #: rejection of null of no cointegration at 1%, 5% and 10% levels respectively. The critical values at these levels are –40.34, –32.33 and –27.78 respectively for  $\hat{Z}_x$  and –4.64, –4.15 and –3.84 respectively for  $\hat{Z}_i$  (see Tables Ic and IIc of Phillips and Ouliaris, 1990).

The sample period in panel (c) is 1889–1990 for USA–UK and 1920–90 for USA–SW and UK–SW. JA: Japan, FR: France, IT: Italy, CA: Canada, SW: Sweden.

TABLE 4. Summary of cointegration test results presented in Tables 2 and 3.

Data set	Park method		Phillips and Ouliaris method	
	(1)	(2)	(3)	(4)
OECD QNA	20/45	33/45	42/45	42/45
IFS	13/18	15/18	15/18	15/18
Historical data	5/9	9/9	9/9	9/9

Notes: (1) Proportion of Park (1990) test statistics that reject the null of cointegration at the 10% level. (2) Proportion of cases where the Park (1990) test statistic rejects the hypothesis of cointegration at the 10% level or for which statistically significant (at the 5% level) violations of the concavity restriction on the utility function ( $\sigma < 1$ ) occur.

(3) Proportion of Phillips and Ouliaris (1990)  $\hat{Z}_x$  test statistics that fail to reject the null of no cointegration at the 10% level.

(4) Proportion of Phillips and Ouliaris (1990)  $\hat{Z}_i$  test statistics that fail to reject the null of no cointegration at the 10% level.

hypothesis fails to be rejected at the 10 percent level by 42 of the 45  $\hat{Z}_\alpha$  test statistics and by 42 of the 45  $\hat{Z}_t$  test statistics reported for the OECD consumption data.

*Results for IFS data on total consumption expenditures*

The null of cointegration is rejected at the 10 percent level by 13 of the 18 Park test statistics reported for the IFS consumption data, while the Phillips and Ouliaris tests show that the null of no cointegration fails to be rejected (at the 10 percent level) by 15 of the 18 reported  $\hat{Z}_\alpha$  test statistics and by 15 of the 18  $\hat{Z}_t$  test statistics.

*Results for historical data on total private consumption*

For the historical data, the null hypothesis of cointegration is rejected at the 10 percent level by 5 of the 9 reported Park test statistics, while statistically significant (at the 5 percent level) rejections of concavity occur in 8 of the 9 cases. None of the Phillips and Ouliaris test statistics obtained for the historical data rejects the null hypothesis of no cointegration.

*IV.C. Results for the high-frequency risk-sharing condition <4>*

To evaluate whether <4> is consistent with the data, the following regressions were run (by OLS) for all country pairs in the three data sets:

<6a>                     $\Delta \ln(c_t^i) = a_1 + b_1 \Delta \ln(c_t^j) + d_1 \Delta \ln(R_t^{i,j}) + \varepsilon_t^1,$   
<6b>                     $\Delta \ln(c_t^j) = a_2 + b_2 \Delta \ln(c_t^i) + d_2 \Delta \ln(R_t^{i,j}) + \varepsilon_t^2,$   
<6c>                     $\Delta \ln(R_t^{i,j}) = a_3 + b_3 \Delta \ln(c_t^i) + d_3 \Delta \ln(c_t^j) + \varepsilon_t^3.$

Table 5, which reports  $R^2$ s for <6a>–<6c>, shows that these regressions do not fit the data well: the  $R^2$ s are generally close to zero. We thus see that for a typical

TABLE 5. Tests of high-frequency implications of complete markets model.

(a) $R^2$ s for regressions <6a>–<6c>			
Quarterly non-durables plus service consumption (OECD QNA, 1971:II–1988:I)			
USA–JA	0.05	0.02	0.02
USA–FR	0.14	0.09	0.05
USA–UK	0.08	0.10	0.03
USA–IT	0.05	0.00	0.05
USA–CA	0.00	0.00	0.00
JA–FR	0.10	0.06	0.07
JA–UK	0.06	0.06	0.01
JA–IT	0.09	0.08	0.04
JA–CA	0.02	0.05	0.03

*continued*

TABLE 5. Continued.

(a) $R^2$ s for regressions $\langle 6a \rangle - \langle 6c \rangle$			
Quarterly non-durables plus service consumption (OECD QNA, 1971:II–1988:I)			
FR–UK	0.07	0.07	0.03
FR–IT	0.02	0.00	0.02
FR–CA	0.02	0.01	0.01
UK–IT	0.07	0.06	0.04
UK–CA	0.05	0.04	0.01
IT–CA	0.09	0.09	0.00
Quarterly total consumption (IFS, 1957:II–1991:II)			
USA–JA	0.10	0.08	0.04
USA–UK	0.05	0.04	0.02
USA–CA	0.12	0.10	0.02
JA–UK	0.02	0.02	0.00
JA–CA	0.00	0.03	0.03
UK–CA	0.03	0.02	0.01
Historical data (annual) on total consumption (1890–1990/1921–90)			
USA–UK	0.00	0.00	0.00
USA–SW	0.00	0.03	0.03
UK–SW	0.29	0.31	0.02

(b)  $R^2$ s for versions of equations  $\langle 6a \rangle - \langle 6c \rangle$  which include growth rates of hours worked in countries  $i$  and  $j$  as additional regressors

Quarterly non-durables plus services consumption (OECD QNA, 1971:II–1988:I)			
USA–JA	0.13	0.09	0.04
USA–FR	0.23	0.10	0.09
USA–UK	0.19	0.27	0.03
USA–IT	0.19	0.07	0.06
USA–CA	0.12	0.09	0.06
JA–FR	0.15	0.10	0.10
JA–UK	0.21	0.24	0.04
JA–IT	0.12	0.13	0.05
JA–CA	0.07	0.05	0.03
FR–UK	0.07	0.27	0.05
FR–IT	0.11	0.08	0.02
FR–CA	0.02	0.04	0.05
UK–IT	0.26	0.11	0.07
UK–CA	0.28	0.05	0.02
IT–CA	0.09	0.14	0.01

Notes: For country pair  $i-j$ , columns 1–3 of panel (a) report  $R^2$ s from regressions  $\langle 6a \rangle - \langle 6c \rangle$  respectively. For country pair  $i-j$ , columns 1–3 of panel (b) report  $R^2$ s from the following regressions respectively:

$$\Delta \ln(c_t^i) = a_1 + b_1 \Delta \ln(c_t^j) + d_1 \Delta \ln(R_t^{i,j}) + e_1 \Delta \ln(L_t^i) + f_1 \Delta \ln(L_t^j) + \varepsilon_t^1,$$

$$\Delta \ln(c_t^j) = a_2 + b_2 \Delta \ln(c_t^i) + d_2 \ln(R_t^{i,j}) + e_2 \Delta \ln(L_t^i) + f_2 \Delta \ln(L_t^j) + \varepsilon_t^2,$$

$$\Delta \ln(R_t^{i,j}) = a_3 + b_3 \Delta \ln(c_t^i) + d_3 \Delta \ln(c_t^j) + e_3 \Delta \ln(L_t^i) + f_3 \Delta \ln(L_t^j) + \varepsilon_t^3.$$

Here,  $L_t^i$  and  $L_t^j$  denote per capita hours worked in countries  $i$  and  $j$  respectively.

JA: Japan, FR: France, IT: Italy, CA: Canada, SW: Sweden.

country pair, only a very small fraction of the movements in the consumption growth rate of one of the countries in the pair can be 'explained' by the contemporaneous growth rate of the other country and by the growth rate of the bilateral real exchange rate.

This shows that the complete markets model cannot match the observed consumption and real exchange rate growth rates, at least not unless substantial taste shifts are assumed.<sup>9</sup> One possible interpretation of the taste shock  $A_t^k$  in the utility function  $\langle 2 \rangle$  is that it captures variables, such as hours worked, that interact non-separably with consumption in the utility function. To examine whether the complete markets model is better able to match high frequency features of the data when movements in hours are explicitly taken into account, growth rates of per capita hours of work in countries  $i$  and  $j$  were included as additional regressors in equations  $\langle 6a \rangle$ – $\langle 6c \rangle$ .<sup>10</sup> Panel (b) of Table 5 reports the  $R^2$ s that were obtained by fitting these extended regressions to the OECD, QNA data.<sup>11</sup> The average value of the  $R^2$ s reported in panel (b) is only 0.11. Thus, one concludes that even when one allows for preferences that are non-separable in private consumption and hours worked, the complete markets model does not capture well the observed behavior of growth rates of consumption and of real exchange rates.

#### IV.D. Interpretation of empirical evidence on complete markets framework

The test results described in this section could be due to the fact that international asset markets are incomplete or that preferences or other aspects of the model are misspecified. They cast strong doubts on the IRBC literature based on complete asset markets, since the key features of the model tested in this section are the ones typically assumed in that literature.

### V. International consumption comovements in a world with unrestricted international trade in debt contracts

As an alternative to models with complete asset markets, recent IRBC research has started to investigate models of the international economy with incomplete asset markets. For example, Kollmann (1990) and Baxter and Crucini (1991) present two-country IRBC models in which only risk-free real bonds can be used for international financial transactions.<sup>12</sup>

A testable implication of an asset markets structure with unrestricted trade in real risk-free bonds is that (appropriately defined) expected intertemporal marginal rates of substitution in consumption are equated across countries. To see why this is so, note that when country  $j$  can freely trade in real risk-free bonds denominated in units of country  $i$ 's consumption good, then optimal behavior by country  $j$  implies that the following Euler condition is satisfied:

$$\langle 7 \rangle \quad (1 + r_t^i) E_t \{ (R_{t+1}^{i,j} / R_t^{i,j}) \beta^j u_{t+1}^j(c_t^j) / u_t^j(c_t^j) \} = 1.$$

Here,  $r_t^i$  denotes the real one-period risk-free interest rate in terms of country  $i$ 's consumption good (an agent who borrows one unit of the country  $i$  good in period  $t$ , has to pay back  $1 + r_t^i$  units of the same good in period  $t + 1$ ), while (as

before)  $R^{i,j}$  denotes the price of one unit of country  $i$ 's consumption good in terms of country  $j$ 's good. In a world with unrestricted trade in real risk-free bonds, the Euler condition  $\langle 7 \rangle$  holds for any country pair  $i - j$ . This implies that expected marginal rates of substitution between units of country  $i$ 's consumption good at dates  $t$  and  $t + 1$  are equated for any country pair  $i - j$ :

$$\langle 8 \rangle \quad E_t\{\beta^i u_{t+1}^i(c_{t+1}^i)/u_t^i(c_t^i)\} = E_t\{\beta^j u_{t+1}^j(c_{t+1}^j)/u_t^j(c_t^j)[R_{t+1}^{i,j}/R_t^{i,j}]\}.^{13}$$

To test this condition, we again assume the iso-elastic utility function defined in  $\langle 2 \rangle$ , but without taste shocks (it appears that  $\langle 8 \rangle$  captures the data well, even if one does not assume taste shocks). The tests considered below thus focus on the following version of  $\langle 8 \rangle$ :

$$\langle 9 \rangle \quad E_t\{\beta^i (c_{t+1}^i/c_t^i)^{\sigma^i-1}\} = E_t\{\beta^j (c_{t+1}^j/c_t^j)^{\sigma^j-1}[R_{t+1}^{i,j}/R_t^{i,j}]\}.^{14}$$

Generalized Method of Moments (GMM) techniques (Hansen, 1982; Cumby *et al.*, 1983) are used to test  $\langle 9 \rangle$ . Table 6 reports GMM test results for the quarterly OECD data on non-durables plus services consumption.<sup>15</sup> In that table, condition  $\langle 9 \rangle$  is tested separately for each pair of countries in the sample (see panel (a)) as well as jointly for several sets of country pairs (panel (b)). The instruments used in these tests are a constant and lagged growth factors of consumption and real exchange rates.<sup>16</sup>

The GMM tests show that  $\langle 9 \rangle$  fails to be rejected at the 10 percent level (or even at the 20 percent level). The concavity restriction  $\sigma < 1$  is violated by roughly a third of the estimates of the risk-aversion parameter  $\sigma$  which are reported in Table 6, but (with few exceptions) these violations are not statistically significant.

TABLE 6. GMM tests of equation  $\langle 9 \rangle$  (non-durables plus services consumption).

	(a) Separate tests of $\langle 9 \rangle$ for each country pair (d.f. = 4)						
	$J$	$p$ -val.	$\hat{\sigma}^1$	$\hat{\sigma}^2$	$R_1^2$	$R_2^2$	$R_3^2$
USA-JA	2.9	0.56	-10.7 (3.7)	-0.9 (3.1)	0.20	0.58	0.32
USA-FR	2.0	0.72	-8.6 (7.8)	-0.7 (5.0)	0.18	0.26	0.25
USA-UK	1.0	0.90	3.4 (8.3)	3.6 (4.1)	0.10	0.13	0.10
USA-IT	1.7	0.77	-1.1 (4.7)	7.2 (7.6)	0.24	0.56	0.23
USA-CA	5.4	0.24	1.3 (1.5)	1.9 (1.4)	0.12	0.11	0.19
JA-FR	2.0	0.72	-3.5 (4.2)	3.1 (3.5)	0.52	0.17	0.20
JA-UK	0.7	0.94	-2.6 (7.3)	-5.4 (11.0)	0.56	0.08	0.19
JA-IT	2.6	0.61	-1.4 (4.3)	3.2 (5.3)	0.53	0.61	0.22
JA-CA	1.2	0.87	-2.1 (5.5)	-9.5 (9.0)	0.54	0.13	0.22
FR-UK	1.6	0.79	-0.9 (3.0)	-1.6 (4.0)	0.20	0.09	0.12
FR-IT	2.2	0.69	-0.8 (2.3)	2.5 (4.1)	0.29	0.59	0.30
FR-CA	1.7	0.79	2.4 (7.6)	-2.6 (9.4)	0.23	0.14	0.28
UK-IT	2.0	0.72	-0.3 (4.8)	-2.4 (7.9)	0.10	0.58	0.14
UK-CA	1.6	0.79	6.4 (5.2)	3.3 (8.2)	0.10	0.08	0.12
IT-CA	2.2	0.68	-2.3 (8.9)	-6.6 (8.5)	0.56	0.08	0.23

*continued*

TABLE 6. Continued.

(b) Joint tests of <9> for sets of country pairs (d.f. = 24)							
(1) Set of country pairs: USA-JA, USA-FR, USA-UK, USA-IT, USA-CA							
$J$	$p$ -val.	$\hat{\sigma}^{\text{USA}}$	$\hat{\sigma}^{\text{JA}}$	$\hat{\sigma}^{\text{FR}}$	$\hat{\sigma}^{\text{UK}}$	$\hat{\sigma}^{\text{IT}}$	$\hat{\sigma}^{\text{CA}}$
20.8	0.64	-6.8 (2.0)	0.6 (1.5)	4.6 (2.9)	1.5 (1.8)	17.1 (5.6)	-4.4 (2.0)
(2) Set of country pairs: JA-USA, JA-FR, JA-UK, JA-IT, JA-CA							
$J$	$p$ -val.	$\hat{\sigma}^{\text{USA}}$	$\hat{\sigma}^{\text{JA}}$	$\hat{\sigma}^{\text{FR}}$	$\hat{\sigma}^{\text{UK}}$	$\hat{\sigma}^{\text{IT}}$	$\hat{\sigma}^{\text{CA}}$
8.9	0.99	-9.8 (3.4)	-2.3 (2.4)	2.3 (2.7)	-8.7 (5.1)	4.8 (1.6)	-6.2 (6.2)
(3) Set of country pairs: FR-USA, FR-JA, FR-UK, FR-IT, FR-CA							
$J$	$p$ -val.	$\hat{\sigma}^{\text{USA}}$	$\hat{\sigma}^{\text{JA}}$	$\hat{\sigma}^{\text{FR}}$	$\hat{\sigma}^{\text{UK}}$	$\hat{\sigma}^{\text{IT}}$	$\hat{\sigma}^{\text{CA}}$
15.9	0.88	-1.7 (2.1)	-4.9 (3.1)	3.8 (1.9)	0.6 (6.1)	10.0 (6.7)	-0.6 (1.8)
(4) Set of country pairs: UK-USA, UK-JA, UK-FR, UK-IT, UK-CA							
$J$	$p$ -val.	$\hat{\sigma}^{\text{USA}}$	$\hat{\sigma}^{\text{JA}}$	$\hat{\sigma}^{\text{FR}}$	$\hat{\sigma}^{\text{UK}}$	$\hat{\sigma}^{\text{IT}}$	$\hat{\sigma}^{\text{CA}}$
20.9	0.64	-2.5 (3.1)	3.1 (2.6)	0.0 (1.1)	0.3 (1.2)	-8.8 (5.9)	-4.0 (3.5)
(5) Set of country pairs: IT-USA, IT-JA, IT-FR, IT-UK, IT-CA							
$J$	$p$ -val.	$\hat{\sigma}^{\text{USA}}$	$\hat{\sigma}^{\text{JA}}$	$\hat{\sigma}^{\text{FR}}$	$\hat{\sigma}^{\text{UK}}$	$\hat{\sigma}^{\text{IT}}$	$\hat{\sigma}^{\text{CA}}$
22.9	0.52	-0.3 (1.8)	-9.3 (4.0)	-2.7 (1.0)	3.8 (3.6)	-2.5 (1.4)	-2.8 (5.8)
(6) Set of country pairs: CA-USA, CA-JA, CA-FR, CA-UK, CA-IT							
$J$	$p$ -val.	$\hat{\sigma}^{\text{USA}}$	$\hat{\sigma}^{\text{JA}}$	$\hat{\sigma}^{\text{FR}}$	$\hat{\sigma}^{\text{UK}}$	$\hat{\sigma}^{\text{IT}}$	$\hat{\sigma}^{\text{CA}}$
20.5	0.66	0.1 (2.7)	3.9 (1.4)	2.9 (2.6)	3.0 (2.8)	-2.0 (2.4)	-2.3 (2.0)

Notes: The sample period is 1972:I-1988:I (quarterly data). The tests use data on non-durables plus services consumption expenditures from the OECD QNA.  $J$ : Hansen's (1982)  $J$ -statistic.  $p$ -val.: the probability value of the  $J$  statistic. d.f.: degrees of freedom.

Denote the first and second country listed for a given country pair by  $i$  and  $j$  respectively (eg, for USA-JA,  $i$  = USA and  $j$  = Japan). The set of instruments used for the country pair  $i$ - $j$  is given by the vector  $Z_t^{i,j} \equiv (1, (c_t^i/c_{t-1}^i), (c_t^j/c_{t-1}^j), (R_t^{i,j}/R_{t-1}^{i,j}), (c_{t-1}^i/c_{t-2}^i), (c_{t-1}^j/c_{t-2}^j), (R_{t-1}^{i,j}/R_{t-2}^{i,j}))'$ . Let  $\eta_{t+1}^{i,j} \equiv (c_{t+1}^i/c_t^i)^{\sigma^1-1} - b^{i,j} (R_{t+1}^{i,j}/R_t^{i,j}) (c_{t+1}^j/c_t^j)^{\sigma^2-1}$ , where  $b^{i,j} \equiv \beta^i/\beta^j$ .

Panel (a) separately tests condition <9> for each country pair in the sample. Specifically, the orthogonality condition tested for country pair  $i$ - $j$  is  $E\{\eta_{t+1}^{i,j} Z_t^{i,j}\} = 0$ .

$\hat{\sigma}^1$ ,  $\hat{\sigma}^2$ : GMM estimates of preference parameters  $\sigma^1$  and  $\sigma^2$  (standard errors in parentheses).  $\sigma^1$  pertains to the first country listed for a given country pair. Note: strict concavity of the period utility function requires  $\sigma < 1$ .

The  $R_1^2$ ,  $R_2^2$ ,  $R_3^2$  coefficients reported for country pair  $i$ - $j$  are  $R^2$  coefficients from OLS regressions of  $(c_{t+1}^i/c_t^i)$ ,  $(c_{t+1}^j/c_t^j)$  and  $(R_{t+1}^{i,j}/R_t^{i,j})$  respectively on the instruments used for that country pair.

Panel (b) considers six different sets of country pairs. For each set, the joint hypothesis that  $E\{\eta_{t+1}^{i,j} Z_t^{i,j}\} = 0$  holds for each of the country pairs included in the set is tested (Note: the first country listed in a given country pair corresponds to country 'i').

$\hat{\sigma}^{\text{USA}}$ ,  $\hat{\sigma}^{\text{JA}}$ ,  $\hat{\sigma}^{\text{FR}}$ ,  $\hat{\sigma}^{\text{UK}}$ ,  $\hat{\sigma}^{\text{IT}}$ ,  $\hat{\sigma}^{\text{CA}}$ : GMM estimates of the ' $\sigma$ ' preference parameter for the USA, Japan, France, UK, Italy and Canada respectively (standard errors in parentheses).

Hence, it appears that growth factors of consumption and real exchange rates behave in a manner which is consistent with the assumption of unrestricted international trade in real risk-free bonds. This finding is closely related to empirical tests presented by Obstfeld (1989) (using different econometric techniques) which suggest that, for the period after 1972, the behavior of

consumption series for the USA, Japan and Germany is largely consistent with free international trade in *nominal* risk-free bonds.

Obviously, the GMM results do not imply that the models developed by Kollmann (1990) and Baxter and Crucini (1991)—which are based on the assumption that *only* debt contracts can be used for international financial transactions—constitute the most appropriate approach for modeling the incompleteness of international asset markets. An interesting alternative to their framework is one where debt contracts and a limited set of state-contingent assets can be traded internationally. Cochrane (1991) and Obstfeld (1993) have recently proposed methods that allow one to determine empirically what specific types of risks (if any) can be pooled through trade in state-contingent assets.

As the structure of asset markets affects the behavior of all variables (not just that of consumption), tests of fully specified models with different restrictions on asset markets would also help to determine which specific incomplete asset markets framework best represents reality. Conducting such tests is left for future research. Fully solved models would also have to be considered in order to analyze (and test) the implications of incomplete asset markets for the long-run properties of the data.

## VI. Summary

With iso-elastic instantaneous utility functions—as are typically assumed in existing international RBC models—and under certain restrictions on the behavior of taste shocks, an international RBC model with complete international asset markets predicts that log consumption and log bilateral real exchange rate series are cointegrated for any pair of countries. The paper tests this prediction using cointegration techniques and data for the USA, Japan, France, UK, Italy, Canada and Sweden. The results suggest that existing international RBC models with complete asset markets fail to adequately capture the trend behavior of consumption and real exchange rates. The risk-sharing made possible by complete markets also implies that, for any pair of countries, a close relationship exists between the growth rates of consumption in these countries and the growth rates of their bilateral real exchange rate. The paper casts doubts on the empirical validity of this relationship between high-frequency consumption and real exchange rate movements as well. It appears however that the behavior of growth factors of consumption and bilateral real exchange rates is consistent with unrestricted international trade in real risk-free bonds.

## Appendix

### *Derivation of equation <1>*

Equation <1> can be derived as follows: Let  $p_t^k(s_{t+1})$  be the date  $t$  price (in terms of  $k$ 's consumption) of an asset that pays one unit of  $k$ 's consumption if and only if the state of the world in  $t + 1$  is  $s_{t+1} \in S_{t+1}$ , where  $S_{t+1}$  is the set of possible states in  $t + 1$ . Optimal consumption behavior by country  $k$  implies that  $\pi_t(s_{t+1})\beta^k u^k(c_{t+1}^k(s_{t+1}))/u^k(c_t^k) = p_t^k(s_{t+1})$  holds for all

$s_{t+1} \in S_{t+1}$  where  $c_{t+1}^k(s_{t+1})$  is  $k$ 's consumption in  $t+1$  if  $s_{t+1}$  obtains, while  $\pi_t(s_{t+1})$  is the probability density of  $s_{t+1}$  conditional on date  $t$  information (see Sargent, 1987, ch. 3.5, for example). In equilibrium, the following arbitrage condition has to hold for all country pairs  $i, j$ :  $p_t^i(s_{t+1})/p_t^j(s_{t+1}) = R_{t+1}^{i,j}(s_{t+1})/R_{t+1}^{j,i}(s_{t+1})$ , where  $R_{t+1}^{i,j}(s_{t+1})$  is the date  $t+1$  real exchange rate (in terms of consumption) between countries  $i$  and  $j$  which obtains when  $s_{t+1}$  is realized. Using the above optimality condition (for  $k = i, j$ ), we get:  $(\beta^i/\beta^j)^q (u^i(c_{t+q}^i)/u^i(c_{t+q}^j))(1/R_{t+q}^{i,j}) = (u^i(c_t^i)/u^j(c_t^j))(1/R_{t+q}^{i,j})$ . As this condition holds for all  $t$  and  $q$ , we see that  $\langle 1 \rangle$  has to hold for some  $\Lambda^{i,j}$  that is time-invariant.

### Unit root and cointegration tests

It will be assumed that logged consumption and bilateral real exchange rate series can be represented as sums of deterministic linear time trends and of mean-zero random variables as in the following model:

$$x_t = a + bt + Z_t,$$

where  $Z_t$  is random.

### The Augmented Dickey–Fuller unit root test

The Augmented Dickey–Fuller (ADF) test tests the null hypothesis that the  $\{x_t\}$  process has a unit root. To apply this test, the following regression equation is fitted by OLS:

$$\langle A1 \rangle \quad \Delta x_t = \alpha + \gamma t + \phi x_{t-1} + \sum_{s=1}^k \varphi_s \Delta x_{t-s} + u_t.$$

The ADF tests the null hypothesis that  $\phi = 0$ . The ADF test statistic is the studentized value of the OLS estimate of  $\phi$ . Critical values for its distribution under the null are tabulated in Fuller (1976). Note that  $k$  lags of  $\Delta x_t$  are included on the right-hand side of equation  $\langle A1 \rangle$  in order to correct for serial correlation in first differences of  $x_t$  (see, *eg*, Campbell and Perron, 1991, p. 154). The linear time trend  $\gamma t$  is included in  $\langle A1 \rangle$  because the deterministic part of  $x_t$  is a linear trend.

### Park's (1990) cointegration test

Park (1990) shows how to test the null hypothesis that a set of variables is cointegrated. To apply Park's test, it is necessary to correct for serial correlation in the residuals of the regression equation  $\langle 5 \rangle$  and in the first differences of the variables included on the right-hand side of that regression (see Park, 1990, p. 117). To do this, the Newey and West (1987) method is used (allowing for 10 autocorrelations). To implement the Park test, 'superfluous' regressors (such as polynomials in the time index  $t$  or computer-generated random walks) are added to the right-hand side of  $\langle 5 \rangle$  (see Park, 1990, p. 120). The tests reported below use  $t^2$ ,  $t^3$  and  $t^4$  as 'superfluous' regressors.

### The Phillips and Ouliaris (1990) cointegration test

Phillips and Ouliaris (1990) present various methods to test the hypothesis that a set of variables is not cointegrated. Their  $\hat{Z}_\alpha$  and  $\hat{Z}_t$  test statistics are used here. To apply these tests, it is necessary to correct for serial correlation in the first differences of the residuals  $\eta_t$  in the regression equation  $\langle 5 \rangle$  (see p. 171 in Phillips and Ouliaris, 1990). The Newey and West (1987) method is used for that purpose (allowing for 10 autocorrelations).



### The data

The consumption measure used in the second data set described in Section III is total private consumption expenditures from the IFS (line 96f.c) deflated using national consumer price indexes (IFS, line 64). IFS consumption series are provided in seasonally adjusted form. Non-durables and services consumption data published by the OECD Quarterly National Accounts are also provided in seasonally adjusted form, with the exception of Japanese consumption and UK consumption (in current prices). These series were seasonally adjusted using the *esmooth* command in the econometrics program RATS. Population figures from the IFS (for the post-World War II period) and from Friedman and Schwartz (1982) and Mitchell (1976) were used to calculate the per capita consumption series.

Quarterly bilateral real exchange rate series were constructed using nominal exchange rate series from the IFS and consumer price indexes (denominated in domestic currency). Price indexes for the non-durables and services consumption variable from the OECD, QNA were constructed by dividing the OECD, QNA series on non-durables and services consumption expressed in current prices by the corresponding series in constant prices.

The third data set mentioned in Section III was constructed by updating the annual consumption and real exchange rate series provided by the Backus and Kehoe (1990) database (I thank David Backus for making this database available to me) and by Friedman and Schwartz (1963, 1982) using data from the IFS. For the USA, UK and Sweden annual consumption data are available for the period 1889–1990. For the country pair USA–UK, the available real exchange rate data allow us to test the theory for the period 1889–1990. For USA–Sweden and UK–Sweden, tests can merely be conducted for the 1920–90 period, because bilateral real exchange rate series for USA–Sweden and UK–Sweden are available (from the sources mentioned above) for this period only.

Definitions and sources of the hours of work series used for Table 5: USA—total number of hours worked in the non-agricultural sector (series LPHMU from Citibase). Japan—total employment in the non-agricultural sector multiplied by average weekly hours worked (from *Bulletin of Labour Statistics*, International Labour Office, ILO). France—total employment in the non-agricultural sector multiplied by average weekly hours worked (from the *Bulletin of Labour Statistics* (ILO) and *Bulletin Mensuel des Statistiques du Travail* (INSEE)). UK—total employment multiplied by average weekly hours worked (from *Employment Gazette, Supplement with Historical Statistics*, 1992). This source only provides annual series for average hours worked. A quarterly hours worked series was obtained by linear interpolation. Italy—non-agricultural hours worked data are not available from the ILO or the OECD, and, hence, figures on total employment in the non-agricultural sector were used for that country (from the *Bulletin of Labour Statistics* (ILO)). Canada—total hours worked (all jobs) (from *Historical Labour Force Statistics* 1991, Statistics Canada). ILO series for Italy and France pertain to the first month of a given quarter. Japanese employment and hours worked series are provided at a monthly frequency. Observations for the second month of a given quarter were used to construct quarterly series. The hours worked series for Japan, France and Italy taken from these sources exhibit seasonality; these series were seasonally adjusted using the *esmooth* command in the econometrics software RATS.

### Notes

1. IRBC models have also been studied by, among others, Dellas (1986), Crucini (1989, 1993), Conze *et al.* (1990), Costello (1990), Finn (1990), Kollmann (1990, 1991, 1993), Stockman and Tesar (1990), Yi (1990, 1993), McCurdy and Ricketts (1991), Reynolds (1991), Cardia (1991), Leiderman and Razin (1991), Mendoza (1991), Backus and Smith (1993), Boileau (1992), Costello and Prashnik (1992), Devereux *et al.* (1992), Head (1992), Schlagenhauf and Wrase (1992), Yi and Sadka (1992), and Schmitt-Grohé (1993).
2. The same asset market structure is also considered by, for example, Cole (1988) and in small open economy models developed by Cardia (1991), Mendoza (1991) and Leiderman and Razin (1991).

3. Mace (1991) and Cochrane (1991) have used micro data in order to test whether individual consumption data are consistent with complete markets, but these authors focus on high-frequency consumption movements. Recently, Obstfeld (1993) and Lewis (1993), as well as Canova and Ravn (1993), have studied international risk-sharing using methods closely related to those of Mace and Cochrane. Overall, these papers also cast doubts on the complete markets model, but they ignore variations in real exchange rates. After the present research was completed, I became aware of work by Backus and Smith (1993) whose tests for international risk-sharing allow for real exchange rate variations. However these authors too focus on high-frequency aspects of the data. Other related studies on international risk-sharing include Obstfeld (1989, 1992) and Brennan and Solnik (1989).
4. A notable exception is Stockman and Tesar (1990) who allow for non-tradable goods.
5. RBC models typically use iso-elastic utility functions, because (in a model with infinitely lived agents) this preference specification yields steady state growth paths for which consumption growth rates and real interest rates are constant (see King *et al.*, 1988).
6. One possible interpretation of this shock is that it captures arguments of the utility function that interact non-separably with consumption in the utility function, but which are not explicitly modeled in the present analysis (for example, hours worked).
7. To illustrate this, set  $A_t = L_t^{-\phi}$  in  $\langle 2 \rangle$  (with  $\phi > 0$ ), *ie*, let  $u_t = (1/\sigma)L_t^{-\phi}c_t^\sigma$ , where  $L_t$  is per capita hours worked in period  $t$ . Then the following risk-sharing condition holds with complete markets:  $(\sigma^i - 1) \ln(c_t^i) - \phi^i \ln(L_t^i) = \ln(\Lambda^{i,j}) + \ln(\beta^j/\beta^i)t + (\sigma^j - 1) \ln(c_t^j) - \phi^j \ln(L_t^j) + \ln(R_t^{i,j})$ . When  $L_t^i$  and  $L_t^j$  are covariance-stationary, then the prediction that  $\ln(c_t^i)$  and  $\ln(c_t^j)$  are cointegrated continues to hold. Statistical tests suggest that, contrary to what is assumed in RBC theory, per capita hours of work in the countries considered in this paper follow unit root processes. With unit roots in (logged) hours, the existence of complete markets implies that  $\ln(c_t^i)$ ,  $\ln(c_t^j)$ ,  $\ln(R_t^{i,j})$ ,  $\ln(L_t^i)$  and  $\ln(L_t^j)$  are cointegrated. This is rejected empirically (these test results are available upon request—to save space. Section IV only presents tests of the prediction that  $\ln(c_t^i)$ ,  $\ln(c_t^j)$  and  $\ln(R_t^{i,j})$  are cointegrated).
8. A possible exception is Japanese consumption: for lag lengths  $k = 0, 2, 3, 4, 5$ , the ADF test statistic yields strong evidence against the unit root hypothesis; it appears, however, that for  $k \geq 10$  (not shown in Table 1) there is little evidence against this hypothesis. Because of this, I do not exclude Japan from the sample.
9. The tests presented in this paper do not assume that countries have identical preferences. Backus and Smith (1993) conduct tests of a complete markets model in which countries have identical risk-aversion coefficients. When  $\sigma^i = \sigma^j$  is assumed, predictions hold that are somewhat stronger than the ones tested in this section, *eg*, then growth rates of consumption ratios are perfectly correlated with real exchange rate growth rates (provided there are no taste shocks). These stronger predictions are rejected for the data used in this study (results available on request).
10. To motivate these regressions, assume that  $A_t = L_t^{-\phi}$  in  $\langle 2 \rangle$ —*ie*, that the utility function is  $u_t = (1/\sigma)L_t^{-\phi}c_t^\sigma$  (with  $\phi > 0$ ), where  $L_t$  is hours worked in period  $t$ . Then the risk-sharing condition  $\langle 4 \rangle$  becomes  $(\sigma^i - 1)\Delta \ln(c_t^i) - \phi^i \Delta \ln(L_t^i) = \ln(\beta^j/\beta^i) + (\sigma^j - 1)\Delta \ln(c_t^j) - \phi^j \Delta \ln(L_t^j) + \Delta \ln(R_t^{i,j})$ . Hence, versions of  $\langle 6a \rangle$ – $\langle 6c \rangle$  that include  $\Delta \ln(L_t^i)$  and  $\Delta \ln(L_t^j)$  as additional regressors should yield a very good fit of the data.
11. No hours data seem to be available for the sample periods covered by the two other data sets, and hence extended regressions that use hours data are presented only in conjunction with the OECD QNA consumption data.
12. The same asset market structure is also considered by Cole (1988) and in small open economy models developed by, *eg*, Cardia (1991), Mendoza (1991) and Leiderman and Razin (1991). Other types of asset market incompleteness are considered by Cole (1988), Conze *et al.* (1990), Kwan (1990), Schlagenhaut and Wrase (1992) and by McCurdy and Ricketts (1991).
13. As pointed out by the referee, an analogy exists between  $\langle 8 \rangle$  and Roll's (1979) prediction that, in efficient commodity markets, changes in real exchange rates are unpredictable (in the absence of taste shocks,  $\langle 8 \rangle$  implies  $E_t R_{t+1}^{i,j} = R_t^{i,j}$  when consumers are risk-neutral).
14. Note that, as equation  $\langle 8 \rangle$  holds in any asset market structure in which risk-free real

bonds are freely traded across countries, it also holds when asset markets are complete (although it obviously does not *require* the existence of complete markets). This is so because, with complete asset markets, intertemporal marginal rates of substitution in consumption are equated across countries on a state-by-state basis (equation <1> implies  $\beta^i(u_{t+1}^i/u_t^i) = \beta^j(u_{t+1}^j/u_t^j)(R_{t+1}^i/R_{t+1}^j)$ ). Equation <9> can be interpreted as a conditional version of the high-frequency complete markets risk-sharing condition <4>: setting  $\Delta K_t^{i,j} = 0$  in <4>, taking antilogs of that expression and applying the conditional expectations operator  $E_{t-1}$  gives an equation that is equivalent to <9>.

15. Tests for the other data sets described in Section III yield results that are very similar to those obtained for the OECD data, but to save space, these additional results are not presented.
16. See Table 6 for the list of instruments. Panel (a) of the table shows that the instruments have reasonably good predictive power for the growth factors of consumption and of the real exchange rate that appear in <9>.

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