Productive Government Purchases and the Real Exchange Rate

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Empirical research documents that an exogenous rise in government purchases in a given country triggers a *depreciation* of its real exchange rate. This raises an important puzzle, as standard macro theories predict an *appreciation* of the real exchange rate. We argue that this prediction might reflect the conventional assumption that government purchases are unproductive. Using a *simple* frictionless model with efficient international risk sharing, we show that the real exchange can depreciate in response to a rise in government purchases, if these purchases increase domestic private sector productivity, and labor supply is highly elastic. Empirically plausible marginal products of government purchases are sufficient to generate this result.

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1. Introduction

A key prediction of neoclassical (micro-founded) open-economy macro models (e.g., Backus et al., 1994) is that a country-specific rise in government purchases *appreciates* the real exchange rate. These models generally assume that government purchases are *non-productive* (do not raise private sector productivity). A rise in home government purchases thus lowers the wealth of home households, who consume less and work and produce more. The consequent rise in the home marginal utility of consumption is accompanied by an *appreciation* of the home real exchange rate if (as widely assumed) consumption risk is efficiently shared internationally, through global trade in complete financial markets. For risk sharing implies that the ratio of home to foreign marginal utilities of aggregate consumption is aligned with the home real exchange rate (relative price of home consumption); see Kollmann (1991, 1995) and Backus and Smith (1993). [1]

However, much recent empirical research (based on structural vector-autoregressions) documents that an exogenous increase in government spending in a given country triggers a persistent *depreciation* of its real exchange rate, while domestic employment, output and consumption increase (e.g., Kollmann, 1998; Dellas et al., 2005; Gali et al., 2007; Ravn et al., 2007; Kim and Roubini, 2008). This raises an important puzzle that we call the *government purchases-real exchange rate puzzle*.

The present paper shows that a *simple* frictionless open economy model with complete financial markets (and, thus, efficient international risk sharing) can solve this puzzle, if government purchases are *productive*, and labor supply is highly elastic. The set-up here is motivated by a vast theoretical and empirical literature which points to productive effects of government purchases; see, e.g., Aschauer (1989), Barro (1990), Turnovsky (1999), Glomm and Ravikumar (1997), Basu (2001), Kamps (2006) and Bom and Ligthart (2009). The logic of our result is straightforward: a rise in *productive* home government purchases raises the home marginal product of labor, which boosts the home labor supply and output; when this supply effect is sufficiently strong, the home household's wealth and thus her (relative) consumption rise; under efficient risk sharing, the rise in relative home consumption is accompanied by a depreciation of the home real exchange rate. We argue that

[1] A real exchange rate appreciation is also predicted by traditional Keynesian models; in those models, an increase in government purchases raises domestic absorption, and goods market clearing requires an appreciation.

an empirically plausible marginal product of government purchases is sufficient to generate this result.

Intuitively, the key to solving the *government purchases-real exchange rate puzzle* has to be a mechanism through which an increase in government purchases raises the supply of domestic goods sufficiently strongly, relative to domestic absorption, as such a supply effect worsens the country's terms of trade and depreciates its real exchange rate. In a complementary contribution, Kollmann (2010) presented a model with *incomplete* financial markets that can also solve the *government purchases-real exchange rate puzzle*, even when government purchases are non-productive. Market incompleteness limits risk sharing, and thus exacerbates the negative wealth effect (for the home household) of a rise in home non-productive government purchases, which strengthens the increase in the home labor supply and output, and thus may depreciate the home real exchange rate. However, this mechanism implies a strong fall in home relative consumption.[2] The present paper shows that the introduction of productive government purchases can solve the *government purchases-real exchange rate puzzle*, in a standard set-up with efficient risk sharing.[3] Importantly, the solution here is consistent with a *positive* response of consumption to a rise in government purchases--empirically, a rise in government purchases tends to raise private consumption, as mentioned above.

In the following section we lay out the model. Section 3 discusses numerical simulations. Section 4 concludes.

2. The Model

2.1. Environment

The model builds on Kollmann (2010). For simplicity, we consider a one-period set-up. There are two ex-ante symmetric countries, Home (H) and Foreign (F). Each country is inhabited by a representative household and a government. The private sector in country i=H,F produces Y_i units of a tradable intermediate good i, using local labor L_i. The country's government is endowed with a beginning-of-period capital stock $\overline{G}$ (e.g.,

[2] Market incompleteness breaks the direct link between the real exchange rate and relative marginal utilities of consumption—thus, a real exchange rate depreciation may be accompanied by a *fall* in relative consumption.

[3] This possibility was conjectured by Basu (2010) in a short comment on Kollmann (2010); but Basu did not provide a formal model to prove that point.

infrastructure facilities); during the period, the government can engage in (net) investment γ_i, and thereby change the capital stock to $G_i=\overline{G}+\gamma_i$. Government (investment) purchases γ_H, γ_F are exogenous random variables whose distributions are symmetric across countries. These purchases are financed using lump-sum taxes. The (cum-investment) government capital stock G_i potentially raise private sector labor productivity. Country *i*'s intermediate good production function is:

$$Y_i = L_i G_i^{\theta}, \text{ with } \theta \geq 0. \tag{1}$$

The introduction of government capital G_i in the production function is motivated by Aschauer (1989), Barro (1990) and Baxter and King (1993). The parameter θ represents the degree of public capital externality.

Country i also produces a non-traded final good Z_i from local and imported intermediate inputs. Country *i*'s final-good production function is:

$$Z_i = [\alpha^{1/\phi}(y_i^i)^{(\phi-1)/\phi} + (1-\alpha)^{1/\phi}(y_j^i)^{(\phi-1)/\phi}]^{\phi/(\phi-1)} \text{ with } j \neq i, \tag{2}$$

where y_j^i is the amount of intermediate good j used in the production of final good *i*; $\phi>0$ is the substitution elasticity between inputs. We assume $0.5<\alpha<1$, i.e., there is a bias in favor of the use of the local input. The country *i* final good is used for private consumption and for investment by the domestic government.

Country i's technologies are operated by competitive firms owned by the local household. The labor market is likewise competitive. Prices thus equal marginal costs. The price of the country *i* intermediate good, p_i, is hence $p_i = W_i G_i^{-\theta}$ where W_i is the country *i* wage rate. The price of the country *i* final good is:

$$P_i \equiv [\alpha(p_i)^{1-\phi} + (1-\alpha)(p_j)^{(1-\phi)}]^{1/(1-\phi)}, \quad j \neq i. \tag{3}$$

Profit maximization by final goods producers implies the following demand functions for intermediates:

$$y_i^i = \alpha(p_i/P_i)^{-\phi} Z_i, \; y_j^i = (1-\alpha)(p_j/P_i)^{-\phi} Z_i \text{ for } j \neq i. \tag{4}$$

The country i household has utility function $U_i = \frac{C_i^{1-\sigma}-1}{1-\sigma} - \frac{L_i^{1+1/\eta}}{1+1/\eta}$, where C_i is private consumption and $\sigma, \eta > 0$ are the risk aversion coefficient and the (Frisch) labor supply elasticity, respectively. The household equates her marginal rate of substitution between consumption and leisure to the real wage rate, W_i/P_i. Thus:

$$C_i^{-\sigma} \frac{p_i}{P_i} G_i^{\theta} = L_i^{1/\eta}. \tag{5}$$

Market clearing requires $Y_i = y_i^H + y_i^F$ and $Z_i = C_i + \gamma_i$ for i=H,F. The Home terms of trade are $q = p_H / p_F$. We define the Home real exchange rate as the price of final good H in units of final good F: $rer \equiv P_H/P_F$; thus, a rise in *rer* is an *appreciation* of the Home real exchange rate.

The timing of decisions is as follows. *Before* government purchases γ_H, γ_F are realized, the households trade in a complete set of Arrow-Debreu securities. Production and consumption takes place after γ_H, γ_F are realized. The existence of complete markets entails that the equilibrium consumption allocation is efficient; hence, in equilibrium, the ratio of Home to Foreign households' marginal utilities of consumption is equated to the real exchange rate (Kollmann, 1991, 1995; Backus and Smith, 1993):

$$(C_H)^{-\sigma}/(C_F)^{-\sigma} = rer.\ ^4 \tag{6}$$

Hence, Home aggregate consumption rises relative to Foreign consumption, in states of the world in which the Home real exchange rate depreciates (i.e., when the Home consumption good becomes cheaper relative to Foreign consumption).

2.2. Model Solution

We assume that the expected value of (net) government investment is zero: $E\gamma_i = 0$, for i=H,F, so that the expected value of the government capital stock is $E(G_i) = \overline{G}$. We linearize

[4] In general, efficient risk sharing implies $(C_H)^{-\sigma}/(C_F)^{-\sigma} = \Lambda \cdot rer$, where Λ is a state-invariant coefficient that reflects the relative initial wealth of the two countries; in the set-up here, Λ=1 holds as the countries are ex ante symmetric,

the model around the equilibrium that obtains when $\gamma_H=\gamma_F=0$, $G_H=G_F=\overline{G}$. $\hat{x}\equiv(x-\bar{x})/\bar{x}$ is the relative deviation of a variable x from the point of linearization, $\bar{x}$. Variables without subscripts are ratios of Home to Foreign variables: $y\equiv Y_H/Y_F$, $c\equiv C_H/C_F$, $g\equiv G_H/G_F$. Equation (3) implies:

$$\widehat{rer}=(2\alpha-1)\hat{q}. \tag{7}$$

Thus, a Home terms of trade improvement induces a real exchange rate appreciation (as $\alpha>0.5$).

Equation (4) implies that relative *world* demand for intermediate good H (compared to demand for good F) is: $d\equiv\frac{y_H^H+y_H^F}{y_F^H+y_F^F}=q^{-\phi}\frac{\alpha rer^{\phi}z+1-\alpha}{\alpha+(1-\alpha)rer^{\phi}z}$, where $z\equiv Z_H/Z_F=(C_H+\gamma_H)/(C_F+\gamma_F)$. Market clearing requires that relative demand equals relative output: $d=y$. This implies:

$$\hat{y}=-\lambda\hat{q}+(2\alpha-1)\Gamma(\gamma_H-\gamma_F)/\overline{G}, \tag{8}$$

where $\lambda\equiv 4\alpha(1-\alpha)\phi+(2\alpha-1)^2/\sigma>0$ and $\Gamma\equiv\overline{G}/\overline{Y}$ with $\overline{Y}\equiv\overline{Y_H}=\overline{Y_F}$. [5]

Note that (8) is an 'effective' relative *demand* function for the Home intermediate good; λ is the elasticity of relative world demand for the Home intermediate good with respect to the Home terms of trade. Relative demand is decreasing in the Home terms of trade. Holding constant the terms of trade q, an increase in Home relative government purchases raises relative demand for the Home good (as $\alpha>0.5$).

The optimal consumption/leisure trade-off (5) and the risk sharing condition (6) imply $\hat{l}=\eta\hat{q}+\eta\theta\hat{g}$, where $l\equiv L_H/L_F$, is the relative country H labor input. The production function (1) implies $\hat{y}=\hat{l}+\theta\hat{g}$. Thus:

$$\hat{y}=\eta\hat{q}+(1+\eta)\theta\hat{g}. \tag{9}$$

Equation (9) is a relative *supply* function of the Home intermediate good. A Home terms of trade appreciation raises the relative home supply. This happens because the terms of trade improvement raises the Home marginal product of labor, in units of final consumption, which increases the Home labor supply. Holding q constant, a rise in Home

[5] To get (8), we use $\hat{z}=\hat{c}+(\gamma_H-\gamma_F)/\overline{Y}$ (as $\overline{Y}=\overline{C_i}$), and the risk sharing condition: $\hat{c}=-\sigma^{-1}(2\alpha-1)\hat{q}$ (see (6), (7)).

government purchases raises the relative supply of the Home intermediate good, if government purchases are productive, $\theta>0$.

(8) and (9) can be solved for the equilibrium real exchange rate (using $\hat{g}=(\gamma_H-\gamma_F)/\overline{G}$):

$$\widehat{rer} = \Psi\ \hat{g}, \quad \text{with } \Psi \equiv -\{\theta/\Gamma - (2\alpha-1)/(1+\eta)\}(2\alpha-1)(1+\eta)/(\lambda+\eta). \tag{10}$$

Note that θ/Γ is the marginal product of government capital (*MPG*), at the point of linearization.[6]

When government capital does not raise private sector productivity ($MPG\equiv\theta/\Gamma=0$), then $\Psi>0$, i.e., a rise in (relative) Home government purchases *appreciates* the Home real exchange rate. A rise in Home government purchases raises the relative demand for the Home intermediate good (see (8)). When *MPG*=0, the rise in government purchases does not affect the (relative) supply of the Home intermediate good; thus, the rise in demand improves the Home terms of trade, and the Home real exchange rate appreciates. Intuitively, it can also be noted that a rise in Home government purchases results in an adverse wealth effect for the Home household; this lowers (relative) Home aggregate consumption, and thus has to be accompanied by a Home real exchange rate appreciation (via the risk sharing condition (6)).

When government purchases are productive ($MPG\equiv\theta/\Gamma>0$), then a rise in government purchases shifts up the (relative) supply of the Home intermediate good (9), which mitigates the terms of trade improvement. When *MPG* is sufficiently high, this supply-side effect dominates and the Home terms of trade and the real exchange *depreciate.* Equivalently, note that when the *MPG* is sufficiently high, then a rise in Home government purchases has a positive wealth effect for the Home household, and hence (relative) Home consumption rises, and (because of the risk sharing condition), the Home real exchange rate depreciates. Specifically (as can be seen from (10)) this is the case when

$$MPG > (2\alpha-1)/(1+\eta). \tag{11}$$

3. Quantitative results

Using a production function approach, Aschauer's (1989) seminal study estimated the elasticity of private output with respect to government capital, θ, at about 0.39, for the US. Bom and Ligthart (2009) conduct a meta-analysis of 67 studies that estimated θ (mainly for

[6] $\partial Y_i/\partial G_i=\theta Y_i/G_i$; thus the MPG at the point of linearization is $\theta\overline{Y}/\overline{G}=\theta/\Gamma$.

OECD countries). Estimates vary widely across studies, but almost all estimates are positive and highly statistically significant. Estimates of s*hort-run* elasticities (that measure the sensitivity of output to a public capital change, within the same year or quarter) are generally lower than estimated *long-run* elasticities. Thus public capital is more productive in the long-run than in the short-run—a fact which the static model here cannot capture (a dynamic model would be needed for that purpose). Bom and Ligthart (Tables 4 and 5) construct meta-estimates that synthesize the 67 studies. Meta-estimates of the short-run elasticity range between 0.04 and 0.17, while long-run elasticity meta-estimates lie between 0.16 and 0.29

Kamps (2006) reports that the ratio of public capital to (annual) GDP is about 50% for most OECD economies, i.e. Γ=0.5. (In 2000, the US public capital to GDP ratio was 50%; the average ratio across 22 OECD countries was 51.4%.) Bom and Ligthart's meta-estimates of θ thus imply a short-run *MPG* in the range between 0.08 and 0.34, and a long-run *MPG* in the 0.32 - 0.48 range.

Recall that, in our model, the *MPG* has to exceed the threshold $(2\alpha-1)/(1+\eta)$ for a rise in government spending to depreciate the real exchange rate.

In major industrialized economies, imports represent about 20% of GDP, which implies α=0.8. Macro-models often assume a labor supply elasticity η of 2 or larger, to generate a realistic volatility of hours worked (e.g., Coeurdacier et al., 2008). Setting α=0.8, η=2 yields a threshold value for the *MPG* equal to 0.2. This threshold is below the *long-run* MPGs implied by Bom and Ligthart's (2009) meta-estimates. Hence, the *long-run* rise in domestic output triggered by an increase in public capital (according to the Bom-Ligthart estimates) is sufficiently strong to depreciate the real exchange rate, in our model. Note that the empirical studies discussed in Section 1 suggest that a rise in government purchases triggers a *persistent* real exchange rate depreciation. A depreciation is also predicted for the upper range of the short-run *MPG*s implied by Bom and Ligthart's findings.

A higher value of η lowers the threshold $(2\alpha-1)/(1+\eta)$ and thus strengthens our model's ability to resolve the *government-spending-real exchange rate puzzle.* When labor supply is highly elastic, the Home (relative) labor supply and output rise more strongly, in response to an increase in productive Home government purchases--which makes it more likely that the real exchange rate depreciates. If the labor supply elasticity is infinite, as in

Hansen's (1985) business cycle model with indivisible labor, then the threshold for *MPG* is zero and thus *any* positive value of *MPG* solves the *government purchases-real exchange rate puzzle*.

We next report predicted quantitative responses of macroeconomic variables to an increase in Home government purchase. We set the government capital-to-GDP ratio at Γ=0.5 and again assume α=0.8, η=2. We set the risk aversion coefficient and the substitution elasticity between Home and Foreign intermediates at σ=2 and ϕ=1.5, respectively. These parameter values are well in the range of empirical parameter estimates for industrialized countries (Kollman, 1996; Coeurdacier et al., 2008).

The Table below reports responses of the real exchange rate, and of relative Home output, labor supply and consumption to a 2% increase in the Home government capital stock (that rise in G_H amounts to purchases worth 1% of Home GDP in the absence of the shock).

When θ=0, the Home real exchange rate appreciates by 0.11%; Home relative output and employment rise by 0.38%, while Home relative consumption declines by 0.06%. Larger values of θ strengthen the response of Home relative output, and induce a rise in Home relative consumption. For example, when θ=0.2, the shock to Home government purchases *depreciates* the Home real exchange rate by 0.11% and raises (relative) output and consumption by 0.82% and 0.06%, respectively.

Responses (in %) to 2% increase in Home government capital stock

θ	$\widehat{rer}$	$\hat{y}$	$\hat{l}$	$\hat{c}$
0	0.11	0.38	0.38	-0.06
0.1	0.00	0.60	0.40	0.00
0.2	-0.11	0.82	0.42	0.06
0.3	-0.23	1.04	0.44	0.11
0.4	-0.34	1.25	0.45	0.17

Note: For the values of θ (output elasticity of public capital) listed in Column 1, the Tables reports responses (in %) of the Home real exchange rate (rer), and of relative Home output (y), relative hours worked (l) and relative consumption (c), to a shock that increases the Home government capital stock by 2% (i.e., by 1% of no-shock Home GDP).

4. Conclusion

In this short paper, we have presented a simple static model of a frictionless world economy in which a rise in government purchases can trigger a real exchange rate *depreciation*, if these purchases increase domestic private sector productivity, and labor supply is highly elastic. The key transmission mechanism in our model works though the interaction between the international risk sharing condition and the positive supply side effect of productive government purchases. Future research should focus on the development of a *dynamic* open economy model with productive government capital, to permit formal empirical testing of the paper's central hypothesis. It would be especially interesting to empirically compare the real exchange rate response to shocks to productive and non-productive government purchases.

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